Landscape Study of Target Space Duality of $(0,2)$ Heterotic String Models

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- **Landscape Study:** R. Blumenhagen, TR
  arXiv:1106.4998 [hep-th]
- **Construction and Analysis of new Calabi-Yau 3-folds**
  B. Jurke, TR (in progress)

Motivation: Heterotic String Model Building

Heterotic string models from monads

For heterotic string models on $\mathbb{R}^{1,3}$ we need a CY 3-fold $\mathcal{M}$ and a holomorphic vector bundle $\mathcal{V}$ over $\mathcal{M}$.

- The structure group of $\mathcal{V}$ ($SU(3),\ SU(4),\ SU(5)$) breaks the gauge group E_8 to a GUT group ($E_6,\ SO(10),\ SU(5)$).
- The massless chiral spectrum can be obtained obtained by cohomology groups of $\mathcal{V},\ \mathcal{V}^*,\ \Lambda^2\mathcal{V},\ \Lambda^2\mathcal{V}^*$.
- Many such $\mathcal{V}$ can be constructed via monads $\rightsquigarrow$ need line bundles as building blocks.
- Since the only ingredients are line bundles, **cohomCalg Koszul** extension can be used to calculate the physical data.

The Monad Construction

The Euler sequence

If $\mathcal{V} = T_{\mathcal{M}}$ is the tangent bundle of $\mathcal{M}$, given by intersections of hypersufaces G_j with degree S_j, is given by the cohomology of the Euler complex

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r} \longrightarrow \bigoplus_{i=1}^{d} \mathcal{O}_{\mathcal{M}}(Q_i) \xrightarrow{\otimes \frac{\partial G_j}{\partial x_i}} \bigoplus_{i=1}^{c} \mathcal{O}_{\mathcal{M}}(S_j) \to 0\,.$$

The monad

More generically $\mathcal{V}$ is the cohomology of a complex

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r_{\mathcal{V}}} \longrightarrow \bigoplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a{}^l} \bigoplus_{l=1}^{\lambda} \mathcal{O}_{\mathcal{M}}(M_l) \to 0\,.$$

The Gauge Linear Sigma Model

Bosonic and superpotential

- Superpotential: Contains superfields $X_i,\ P_l$ and $\Gamma_j,\ \Lambda^a$ charged under a $U(1)^r$ gauge group and homogeneous functions G_j and ${F_a}^l$

$$\mathcal{W} = \sum_j \Gamma_j\, G_j(X_i) + \sum_{l,a} P_l\, \Lambda^a\, {F_a}^l(X_i)\,. \tag{1}$$

- Besides the superpotential there is a potential for the bosonic components $x_i,\ p_l$ of the chiral fields $X_i,\ P_l$:

$$V = V_D(x_i, p_l) + V_F(x_i, p_l)\,.$$

  V_D contains r Fayet-Iliopoulos parameter $\xi^{(\alpha)} \in \mathbb{R}$

- The minimum of V,

$$\{(x_i, p_l):\ V = 0\}$$

  has different solutions for different choices of $\xi^{(\alpha)}$.

Phases of the Gauge Linear Sigma Model

Vacuum configuration at the minimum of $V = V_F + V_D$

- Geometric phase e.g. if all $\xi^{(\alpha)} > 0$, vacuum $V = 0$ corresponds to a complete intersection Calabi-Yau space in a toric variety along with holomorphic vector bundle.

Fields & homomgeneous functions in the GLSM define the monad

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r_V} \longrightarrow \bigoplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes {F_a}^l} \bigoplus_{l=1}^{\lambda} \mathcal{O}_{\mathcal{M}}(M_l) \to 0$$

- x_i: homogeneous coordinates of the toric variety $\mathcal{X}$.
- $\{G_j = 0\ \forall j\} = \mathcal{M} \subset \mathcal{X}$ compl inters of hypersurfaces.
- Charges of superfields Λ_a / P_l determine the line bundle degrees: $||\Lambda_a|| = N_a \quad ||P_l|| = -M_l$.
- ${F_a}^l$ bundle defining polynomials.
- Need to satisfy constraints that prevent anomalies:

$$c_1(T\mathcal{M}) = 0, \qquad c_2(T\mathcal{M}) = c_2(\mathcal{V}).$$

Phases of the Gauge Linear Sigma Model

In every phase some fields obtain a vev

- For a generic super potential

$$\mathcal{W} = \sum_j \Gamma_j \, G_j(X_i) + \sum_{l,a} P_l \, \Lambda^a \, {F_a}^l(X_i), \tag{2}$$

  in certain phases (= choices of $\xi^{(\alpha)}$) it happens that a chiral field i.e. P_1 is not allowed to vanish at the corresponding vacuum $V = 0$ and hence has a vev.

- We may then drop P_1 in an effective superpotential and in a certain region of the moduli space we see that e.g. Γ_j, Λ^1 and Λ^2 appear on an equal footing.

- One cannot tell which of the homogeneous functions $\{G_j, {F_1}^1, {F_2}^1\}$ originated from a hypersurface equation and which from defining the bundle.

Two Models Share the Same Phase

G's and F's are indistinguishable

- $\rightsquigarrow$ there is a model that has precisely this phase but with G's and F's interchanged. That is the one we are interested in!

$$0 \to \mathcal{O}_{\tilde{\mathcal{M}}}^{\oplus r_V} \longrightarrow \bigoplus_{a=1}^{\delta} \mathcal{O}_{\tilde{\mathcal{M}}}(\tilde{N}_a) \xrightarrow{\otimes \tilde{F}_a{}^l} \bigoplus_{l=1}^{\lambda} \mathcal{O}_{\tilde{\mathcal{M}}}(M_l) \to 0\,,$$

where

$$\tilde{\mathcal{M}} := \left\{ F_1{}^1 = F_2{}^1 = G_j = 0, \quad \forall j > 2 \right\}\,,$$

$$\tilde{F}_1{}^1 := G_1,\ \tilde{F}_2{}^1 := G_2\,.$$

In order to avoid anomalies in the dual model

$$||F_1{}^1|| + ||F_2{}^1|| = ||G_1|| + ||G_2||\,.$$

Effect of the Exchange

Are the two models dual?

- Exchanging their roles $\rightsquigarrow$ completely new model

$$(\mathcal{M}, \mathcal{V}) \rightsquigarrow (\tilde{\mathcal{M}}, \tilde{\mathcal{V}}) \, .$$

  Geometrically: New Calabi-Yau $\tilde{\mathcal{M}}$ and new bundle $\tilde{\mathcal{V}}$ on $\tilde{\mathcal{M}}$
  Remark: Starting with the tangent bundle will lead to a model that is not the standard embedding!
- First observed by Distler and Kachru for common LG phase.
- We extended this analysis to more general non-geometric phases.
- The models agree in the specific phase which allows for two interpretations:
  1. There is a transition between two different models.
  2. The two models are isomorphic, i.e. dual descriptions of the same thing.

Evidence for a Duality

Is there an isomorphism?

- Necessary conditions for a duality are:
  1. Matching of the chiral spectrum of both models.

     $$h^i(\mathcal{M};\Lambda^k\mathcal{V}) = h^i(\tilde{\mathcal{M}};\Lambda^k\tilde{\mathcal{V}})\,.$$

     For bundles with $SU(3)$-structure:

     $$h^\bullet(\mathcal{M};\mathcal{V}) = h^\bullet(\tilde{\mathcal{M}};\tilde{\mathcal{V}})\,.$$

  2. Matching of the full moduli spaces:

     $$h^{1,1}_{\mathcal{M}} + h^{1,2}_{\mathcal{M}} + h^1(\mathcal{M};\mathrm{End}(\mathcal{V})) = h^{1,1}_{\tilde{\mathcal{M}}} + h^{1,2}_{\tilde{\mathcal{M}}} + h^1(\tilde{\mathcal{M}};\mathrm{End}(\tilde{\mathcal{V}}))$$

     in case that there are no obstructions (see talk of Lara)

Example

Initial model

$$S = \mathbb{P}^5[4,2] \text{ and } \mathcal{V} = T\mathcal{M} \text{ the tangent bundle}$$

$$\begin{aligned} h^\bullet_{\mathcal{M}}(T\mathcal{M}) &= (0,89,1,0) \\ h^{1,1}_{\mathcal{M}} + h^{2,1}_{\mathcal{M}} + h^1_{\mathcal{M}}(\text{End}(T\mathcal{M})) &= 1 + 89 + 190 = 280\,. \end{aligned}$$

Dual model

$$\tilde{\mathcal{M}} = \mathbb{P}^5 \times \mathbb{P}^1 \begin{bmatrix} 0 & 1 & 1 \\ 4 & 1 & 1 \end{bmatrix} \text{ and } \tilde{\mathcal{V}} \text{ given by}$$

$$0 \to \mathcal{O}_{\tilde{\mathcal{M}}} \to \bigoplus_{a=1}^4 \mathcal{O}_{\tilde{\mathcal{M}}}\binom{0}{1} \oplus \mathcal{O}_{\tilde{\mathcal{M}}}\binom{1}{0} \oplus \mathcal{O}_{\tilde{\mathcal{M}}}\binom{0}{2} \to \mathcal{O}_{\tilde{\mathcal{M}}}\binom{0}{4} \oplus \mathcal{O}_{\tilde{\mathcal{M}}}\binom{1}{2} \to 0$$

$$\begin{aligned} h^\bullet_{\tilde{\mathcal{M}}}(\tilde{\mathcal{V}}) &= (0,89,1,0) \\ h^{1,1}_{\tilde{\mathcal{M}}} + h^{2,1}_{\tilde{\mathcal{M}}} + h^1_{\tilde{\mathcal{M}}}(\text{End}(\tilde{\mathcal{V}})) &= 2 + 86 + 192 = 280\,. \end{aligned}$$

What we did

We

- applied the proposed procedure to generate potentially dual models to a list of $(2,2)$ models,
- performed some necessary crosschecks for smoothness,
- calculated the chiral spectrum and dim of the moduli space for the $(0,2)$ model using **cohomCalg Koszul** extension and
- found agreement in a great number of examples!

Lists we scanned:

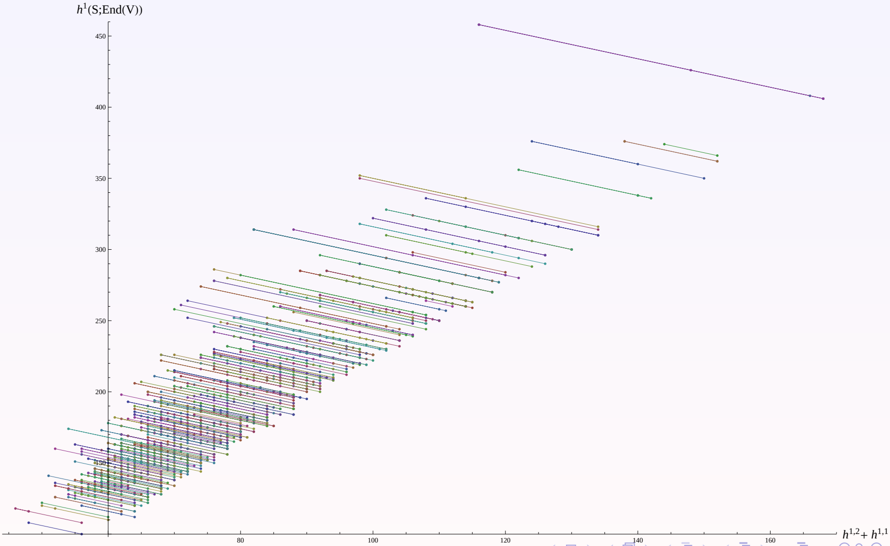
We scanned through two kinds of space:

- Calabi-Yau hypersurfaces in toric varieties with $7, 8, 9$ lattice-point polytopes [Kreuzer, Skarke].
- Part of the list of codim 2 complete intersections in weighted projected spaces [Klemm, Kreuzer, Riegler, Scheidegger].

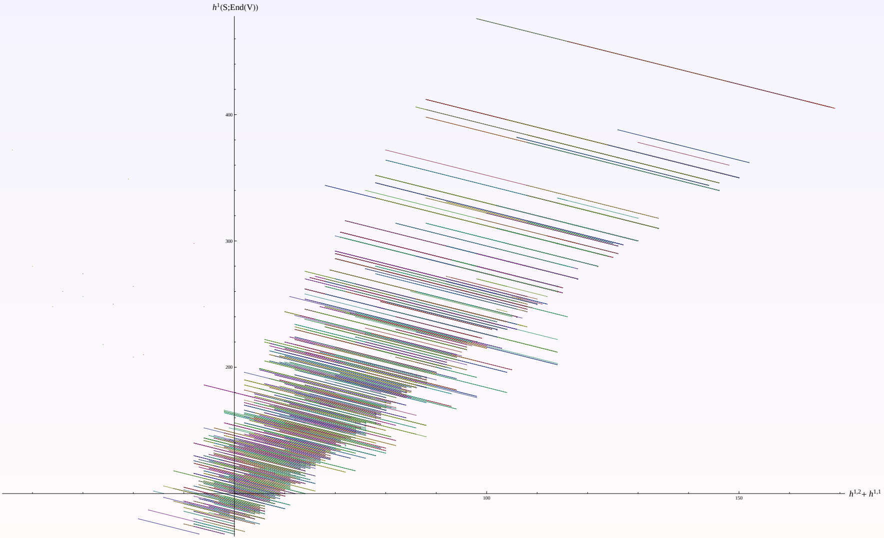
Statistics

Different classes	Possibly smooth models	Models with matching spectrum	Models with full agree-ment	Computed (different) line bdle cohom.
Hypersurfaces as initial space				
1,085	4,507	4,144 (100%)	1509 (95%)	(1,481,539) 3,069,067
Codim 2 complete intersecitons as initial space				
16,029	82,104	67,086 (87%)	20,450 (91%)	(38,807,002) 109,228,732

Plot for Dual Models Starting with Hypersurface Calabi-Yaus



Plot for Dual Models Starting with Codimension two Calabi-Yaus



Conclusions & Outlook

Concluding, we presented

- A proposal how to systematically generate (potentially dual) $(0,2)$-models from given $(0,2)$ or $(2,2)$ models.
- A prove that the anomaly cancellation conditions

$$c_1(T\mathcal{M}) = 0, \qquad c_2(T\mathcal{M}) = c_2(\mathcal{V})$$

  are peserved performing this process.
- An analysis of more than 80,000 different models that provides evidence for a duality rather than a transition.

Outlook and further analysis

- Further look into obstrucions of the moduli space.
- Analysis of stability of th dual bundel (assumed so far).
- A sufficient check for singularities.

Thank you!