Higher derivative couplings on D-branes from T-duality

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University of Texas, February 16, 2010

Based on:

- K. Becker, G. Guo, and D. R., 1007.0441, 1106.????, ...

M-Theory on a Calabi-Yau four-fold

Becker-Becker, Dasgupta-Rajesh-Sethi, many others

Consider M-theory on a Calabi-Yau four-fold with fluxes turned on. Demanding SUSY, one of the equations of motion (the tadpole for the three-form along $\mathbb{R}^{2,1}$) from the low-energy effective theory is

$$d * df = G_4 \wedge *G_4 + \sum_i \delta^{(8)}_{M2}(x - x_i)$$

Here f is some function on the four-fold. Integrating over CY_4, we get

$$0 = \int |G_4|^2 + N_{M2}$$

and we might conclude that M2-branes and non-zero fluxes are forbidden in SUSY solutions.

Including higher-derivative terms

This would not be correct!

The action is known to receive certain higher-derivative corrections, so the equation of motion before becomes

$$d * df = G_4 \wedge *G_4 + \sum_i \delta^{(8)}_{M2}(x - x_i) + \ell_p^6 X_8 + \cdots$$

where X_8 is a particular eight-form built out of four curvature tensors. Locally, X_8 hardly matters, $X_8 \sim 1/R^8$, but integrating we have

$$0 = \int |G_4|^2 + N_{M2} - \frac{\chi(CY_4)}{24}$$

Fluxes go from being **forbidden** to being **required** when we include higher derivatives!

IIB Language

If the CY_4 is elliptically fibered, we can take the fiber to zero size and get a version of the story for IIB on the base B of the fibration (F-theory on the four-fold),

$$0 = \int_B F_3 \wedge H_3 + N_{D3} - \frac{\chi(CY_4)}{24}$$

Puzzle: Where does the $\chi/24$ come from in the IIB description?

IIB doesn't receive corrections until eight-derivatives, but integrating these over the six-dimensional B would give a vanishing contribution in the large volume limit.

But we also have D7/O7 wrapping four-cycles of B, and then a four-derivative correction to the brane action would do the job.

Moral of the story

Indeed,

$$S_{D7} \supset T_7 \frac{\pi^2 \alpha'^2}{24} \int_{D7} C_4 \wedge (\mathrm{tr}\, R_T \wedge R_T - \mathrm{tr}\, R_N \wedge R_n)$$

Moral: Understanding higher-derivative couplings to D-brane actions is necessary to correctly judge the consistency of vacua.

T-dualizing the known corrections

Becker-Guo-Robbins

But these $\mathrm{tr}(R^2)$ corrections to D-brane actions are not the only ones! There will be many others at this same derivative order, involving background fields other than the metric. One way to see this is from T-duality. Consistency of the known couplings with T-duality implies couplings

$$\delta S_{Dp} \sim (\alpha')^2 \int \left\{ C^{(p-3)} \wedge X_4 + C_i^{(p-1)} \wedge X_3^i + C_{ij}^{(p+1)} \wedge X_2^{ij} \right\}$$

where X_4 is a four-form that contains the original $\mathrm{tr}(R^2)$ as well as terms $\sim \partial^2 B \partial^2 B$, $X_3^i \sim \partial^2 h \partial^2 B$ is a three-form along the brane with one normal index, and $X_2^{ij} \sim \partial^2 h \partial^2 h, \partial^2 B \partial^2 B$ is a two-form with two normal indices. Here we performed a linearized (e.g. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$) analysis.

Checking results with disc amplitudes

Becker-Guo-Robbins, Garousi

These couplings too are far from being the whole story. One path to try and determine them would be to demand invariance under diffeomorphisms and gauge transformations and consistency with nonlinear T-duality, but this gets messy quickly.

Another route is to use world-sheet techniques to compute the amplitudes for scattering of closed string fields off the D-brane. This gives only on-shell information, and involves a layer of processing before we can extract the desired couplings.

The two techniques work best in concert, but in the rest of the talk today I will focus on the latter approach and techniques that we have been developing (or rediscovering) to do the computations.

Steps to develop

- Construct the physical state operators
- Construct the boundary state
- Show that amplitudes built from these ingredients are independent of irrelevant choices
- Compute the amplitudes
- Compare to field theory predictions (and thus constrain the action)

Vanishing theorems and picture changing

Frenkel-Garland-Zuckerman, Lian-Zuckerman, Berkovits-Zwiebach

To construct physical state operators, we need to compute the *semi-relative BRST cohomology*, which starts with states annihilated by the operator $b_0 - \widetilde{b}_0$, and then takes the cohomology with respect to the total left- plus right- BRST charge. We will be interested in the degree two cohomology.

We can restrict to states that have well-defined conformal weights, picture charges, and momentum. And it's easy to show that the cohomology is trivial unless the left- and right- conformal weights are zero.

Using results from Lian and Zuckerman, in particular pictures and for nonzero momentum we can always pick a representative with ghost numbers $(1, 1)$ which is separately annihilated by Q and $\widetilde{Q}$.

Using Berkovits and Zwiebach, again at nonzero momentum, we can show that this result holds in all pictures.

Boundary states

We will make use of the boundary state formalism. To compute a disc amplitude with specific boundary conditions we start on the sphere and insert a state

$$|B\rangle$$

which implements the boundary conditions, schematically

$$\Psi\,|B\rangle = \widetilde{\Psi}\,|B\rangle$$

and propagate it outwards to finite radius.

We develop methods for doing computations involving the boundary state without ever needing to write down an explicit form for the boundary state.

Decoupling of BRST exact states

Physically, the amplitudes we construct in this way had better be independent of certain choices we made: the gauge choice (which BRST representative was used for each operator) and the distribution of picture charges among the operators.

Both these statements require the decoupling of BRST-exact states from the amplitude which includes the boundary state. On the sphere, the proofs are very easy, since the BRST charges annihilate everything in sight, but for disc amplitudes this doesn't quite work and instead we get boundary terms, e.g.

$$\left\langle V(z_1, \bar{z}_1) \left\{ Q + \widetilde{Q}, \Lambda(z_2, \bar{z}_2) \right\} \right\rangle_{D_2}$$
$$\to \int_0^{2\pi} d\theta \left\langle V(re^{i\theta} z_1 / z_2, re^{-i\theta} \bar{z}_1 / \bar{z}_2) \Lambda(re^{i\theta}, re^{-i\theta}) | B \right\rangle \Big|_{r=1}^{r=\infty}.$$

Decoupling for generic momenta

$$\left\langle V(z_1,\bar{z}_1)\left\{Q+\widetilde{Q},\Lambda(z_2,\bar{z}_2)\right\}\right\rangle_{D_2}$$
$$\to\int_0^{2\pi}d\theta\left\langle V_1(re^{i\theta}z_1/z_2,re^{-i\theta}\bar{z}_1/\bar{z}_2)\Lambda(re^{i\theta},re^{-i\theta})|B\right\rangle\Big|_{r=1}^{r=\infty}.$$

if V and Λ carry generic momentum, then we believe the amplitude should be analytic in momenta. Near $r=\infty$, the r dependence above will be approximately

$$\sim r^{-2p_1\cdot p_2+a}$$

while near $r=1$,

$$\sim (r-1)^{-2p_1^\perp\cdot p_1^\perp+b}$$

where a and b are some constants which depend on the details. We can continue to a region where the boundary terms vanish, and by analyticity this should hold everywhere.

Comparing with field theory

Putting all these ingredients together, we can compute the amplitudes with confidence. Unfortunately, there is still a layer of processing to compare with the effective action. Because the disc amplitude calculates all field theory contributions at once:

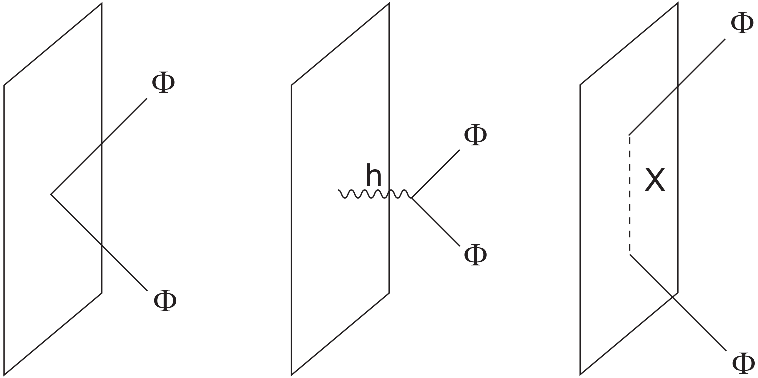

Figure: Dilaton two point function.

Work in progress

We are currently computing the full four-derivative action to third order in the closed string fields and checking the consistency of the result in various ways (comparison to field theory, consistency with T-duality). Hopefully the result can then be rewritten in a more globally nice way, possibly related to twisted K-theory in the same way that the $\mathrm{tr}(R^2)$ terms could be written as

$$\int_X C e^B \sqrt{\frac{\widehat{A}(TX)}{\widehat{A}(NX)}}$$

which can be argued from the untwisted K-theory pairing between the D-brane and the R-R field.