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String instantons, loops and S-duality

- P. Camara E.D, in progress
- P. Camara, E.D., T. Maillard, G. Pradisi, Nucl.Phys.B795:453-489,2008. e-Print: arXiv:0710.3080 [hep-th]

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- Threshold corrections and effective action : heterotic side
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june 1, 2008, String Pheno08

Philadelphia

1. Motivations

$SO(32)$ heterotic -type I duality (Polchinski,Witten) was explored extensively over the last ten years.

- heterotic α' corrections $\rightarrow$ E1 instanton corrections
- NS5 effects $\rightarrow$ E5 instanton corrections

There are simple examples of dual pairs using Vafa-Witten adiabatic argument : freely-acting orbifolds (see Bianchi, Blumenhagen talks).

S-duality allows exact computation of E1 instanton effects in type I : α' corrections on the heterotic side.

Ex : higher-derivative in $N = 4$ SUSY case (Bachas, Kiritsis et coll.)

N=2 case discussed in (Antoniadis,Bachas,Fabre,Partouche and Taylor; Bianchi, Morales).

N=1 models : CDMP, Blumenhagen, Schmidt-Sommerfeld

We will mostly focus today on a standard Z_2 orbifold

- type I side constructed by Bianchi-Sagnotti and Gimon-Polchinski
- heterotic dual pair identified by Berkooz,Leigh,Polchinski,Schwarz, Seiberg,Witten

Our goal : computation of the full one-loop + E1 instanton corrections to the type I effective action .

An important consistency check : comparison of the effective theory Kaplunovsky-Louis formula

$$\frac{4\pi^2}{g^2(\mu)} = Ref_a + \frac{b_a}{4} \ln \frac{M_P^2}{\mu^2}$$
$$+\frac{c_a}{4} K + \frac{C_2(G)}{2} \ln \frac{4\pi^2}{g^2(\mu)} - \sum_r \frac{T_a(r)}{2} \ln det Z_r$$

where $T_a(r)$ is the Dynkin index for the matter representation r with wavefunctions Z_r, and

$$b_a = \sum_r T_a(r) - 3C_2(G) \ , \ c_a = \sum_r T_a(r) - C_2(G)$$

with the one-loop string computation

$$\frac{4\pi^2}{g_a^2(\mu)} = \frac{4\pi^2}{g_{a,0}^2} + \Lambda_a \quad , \text{where}$$

$$\Lambda_a = \frac{b_a}{4} \ln \frac{M_s^2}{\mu^2} + \frac{\Delta_a}{4}$$

Δ_a encodes string threshold corrections from massive string states.

2. The S-dual orbifold pairs

Simplest orbifold $T^4/Z_2 \times T^2$, sixteen fixed points.

Type I side

- One twisted hypermultiplet per fixed point
- Maximal gauge group $U(16)_9 \times U(16)_5$.

Hypers in $(\mathbf{120} + \overline{\mathbf{1}}\mathbf{20}, \mathbf{1}) + (\mathbf{1}, \mathbf{120} + \overline{\mathbf{1}}\mathbf{20}) + (\mathbf{16}, \mathbf{16})$

In order to have a perturbative heterotic dual, distribute 1/2 D5 brane per fixed point.

D5 gauge group broken to $U(1)^{16}$. Each $U(1)$ gets mixed with twisted four forms and become massive.

D9 spectrum : hypers in $\mathbf{120} + \overline{\mathbf{1}}\mathbf{20} + 16 \times \mathbf{16}$.

Heterotic dual

$SO(32)$ compactified to 4d on $T^4/Z_2 \times T^2$. Shift vector on the gauge lattice

$$V = \frac{1}{4}(1, \cdots 1, -3)$$

Gauge group $U(16)$. Charged matter :

untwisted : $\mathbf{120} + \overline{\mathbf{1}}\mathbf{20}$

twisted : $16 \times \mathbf{16}$.

3. Effective action and quantum corrections : type I side

Threshold corrections to gauge couplings depend on moduli of T^2 :

$$T = \frac{\sqrt{G}}{\lambda} + ib \quad , \quad U = \frac{\sqrt{G} + iG_{12}}{G_{22}}$$

One finds (Bachas-Fabre, Antoniadis,Bachas,E.D.)

$$\frac{4\pi^2}{g_a^2(\mu)} = \frac{4\pi^2}{g_{a,0}^2} + \Lambda_a \quad \text{where}$$

$$\Lambda_a = -\frac{1}{4}\tilde{b} \ \ln[\sqrt{G}\mu^2 ReU|\eta(iU)|^4]$$

The effective action is then

$$
\begin{aligned}
f_{D9} &= S + \tilde{b} \ln[ReU\eta^2(iU)] + f_{np} \\
K &= -\ln(S+\bar{S}) - \ln(U+\bar{U}) - \ln(T+\bar{T}) + \cdots \\
&- \frac{4\pi}{3} \frac{E(iU,2)}{(T+\bar{T})(S+\bar{S})} + K_{np}
\end{aligned}
$$

where $E(U,k)$ is the Eisenstein series

$$
E(U,k) \equiv \frac{1}{\zeta(2k)} \sum_{(j_1,j_2)\neq(0,0)} \frac{(\text{Im } U)^k}{|j_1 + j_2 U|^{2k}}
$$

Last term in K needs a separate one-loop computation (ABFPT ; Berg,Haack,Kors). We expect E1 instantonic contributions

$$
f_{np} = \sum_{n=1}^{\infty} g_n(U)\, e^{-2\pi n T} \quad , \quad K_{np} = \sum_{n=1}^{\infty} h_n(U, T+\bar{T})\, e^{-2\pi n T}
$$

4. Effective action and quantum corrections : heterotic side

Threshold corrections are given by (Kaplunovsky)

$$\Lambda_a = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{i}{4\pi} \frac{1}{|\eta|^4} \sum_{\alpha,\beta=0}^{1/2} \partial_\tau \left(\frac{\vartheta \left[{\alpha \atop \beta} \right]}{\eta} \right) \left(Q_a^2 - \frac{1}{4\pi\tau_2} \right) C \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \ ,$$

Q_a is the charge operator of the gauge group; $C \left[{\alpha \atop \beta} \right]$ is the internal six-dimensional partition function.

For the S-dual of the BSGP model we find

$$\Lambda = -\frac{1}{8} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \widehat{Z}_1 \widehat{\mathcal{A}}_f$$

with

$$\hat{\mathcal{A}}_f = -\frac{1}{20\eta^{24}}(D_{10}E_{10} - 48\eta^{24})$$

and

$$\hat{Z}_1 = \frac{\text{Re } T}{\tau_2} \sum_{n_1,\ell_1,n_2,\ell_2} \exp\left[-2\pi T \text{det}(A) - \frac{\pi(\text{Re } T)}{\tau_2(\text{Re } U)} \left| \begin{pmatrix} 1 & iU \end{pmatrix} A \begin{pmatrix} \tau \\ -1 \end{pmatrix} \right|^2 \right.$$

$$A = \begin{pmatrix} n_1 & \ell_1 \\ n_2 & \ell_2 \end{pmatrix} ,$$

with n_i and ℓ_i integers. We used methods of Dixon,Kaplunovsky, Louis and Bachas,Kiritsis et al. to evaluate Λ.

We find

$$\Lambda = \frac{\pi}{2}\text{Re } T_1 - 3\log[(\text{Re } U_1)(\text{Re } T_1)\mu^2|\eta(iU_1)|^4] - \frac{\pi}{3}\frac{E(iU_1,2)}{T_1+\bar{T}_1} -$$

$$-\frac{1}{8}\sum_{k>j\geq 0, p>0}\frac{1}{kp}e^{-2\pi pkT_1}\left[\hat{\mathcal{A}}_f(\mathcal{U}) + \frac{1}{\pi kp}\frac{\hat{\mathcal{A}}_K(\mathcal{U})}{T_1+\bar{T}_1}\right] + \text{c.c.}$$

$\hat{\mathcal{A}}_K$ the almost-holomorphic modular form,

$$\hat{\mathcal{A}}_K = \frac{1}{12\eta^{24}}(\hat{E}_2E_4E_6 + 2E_6^2 + 3E_4^3)$$

We have expressed the result in terms of the induced worldvolume complex structure in the $E1$ multi-instantons wrapping the first 2-torus

$$\mathcal{U} = \frac{j + ipU_1}{k}$$

Split between analytic and non-analytic terms ; the corrected Kähler potential and gauge kinetic function are

$$K = -\log(S+\bar{S}) - \sum_{i=1}^{3} \log[(T_i+\bar{T}_i)(U_i+\bar{U}_i)] + \frac{1}{8\pi^2}\frac{V_{1-loop}+V_{E1}}{S+\bar{S}},$$

$$V_{1-loop} = -\frac{4\pi}{3}\frac{E(iU_1,2)}{T_1+\bar{T}_1} \ , \tag{1}$$

$$V_{E1} = -\sum_{k>j\geq 0,\ p>0} \frac{e^{-2\pi kpT_1}}{4\pi^2(kp)^2}\left[\frac{\widehat{\mathcal{A}}_K(\mathcal{U})}{T_1+\bar{T}_1} - \frac{\pi kp}{2Re\mathcal{U}}\frac{E_{10}(\mathcal{U})}{\eta^{24}(\mathcal{U})}\right] \ + \ \text{c.c.} \ ,$$

$$f_{U(16)} = S \ + \ 12\log\ \eta(iU_1) - \frac{1}{4}\sum_{k>j\geq 0,\ p>0}\frac{e^{-2\pi kpT_1}}{kp}\mathcal{A}_f(\mathcal{U}),$$

where the holomorphic modular forms $\mathcal{A}_f$ and $\mathcal{A}_K$, are defined as before, replacing $\widehat{E}_2$ by E_2.

5. Instantonic corrections in Type I :comments

S-duality $\rightarrow$ E1 corrections on type I side

$$f_{np} \; = \; -\frac{1}{4} \sum_{k>j\geq 0, \; p>0} \frac{e^{-2\pi k p T_1}}{kp} \mathcal{A}_f(\mathcal{U})$$

Comments :

They can be important for :

1) moduli stabilization (CDMP)

Suppose S, U moduli are stabilized and there are field-theory (E5) nonperturbative effects on D9 branes (gaugino condensation, racetrack). Then

$$W_{np} \; = \; A e^{-B(f+f_{np})} = A' \sum_{n=1}^{\infty} d_n e^{-2\pi n T}$$

• It can generate split SUSY like spectra:

$$M_{1/2} \sim e^{-2\pi T} m_{3/2}$$

provided no magnetic fields on D9 branes and no instantonic mass terms for fields charged under SM, $\exp(-2\pi T)Q\tilde{Q}$.

• We found also the perturbative one-loop correction to the Kahler.

• Similar to the perturbative threshold corrections , the E1 instantonic corrections to the gauge kinetic functions from $N = 2$ sectors have some universal properties. Similar results therefore for $N = 1$ models with

$N = 2$ sectors : Z_6, Z_6', etc.

- We found the E1 corrections to the Kahler potential.

They can also be of interest for moduli stabilization and SUSY breaking.

6. Comments about instantonic prefactors

It is possible, by S-duality, to compute explicitly instantonic corrections to the gauge kinetic functions and their dependence on the properties of the gauge theory (P.Camara,E.D.).

Our example: $\mathbb{Z}_2 \times \mathbb{Z}_2$ freely-acting orbifold

We consider a type I $\mathbb{Z}_2 \times \mathbb{Z}_2$ $\mathcal{N} = 1$ family of models

$$(x^1, x^2, x^3, x^4, x^5, x^6) \xrightarrow{g} (x^1 + 1/2, x^2, -x^3, -x^4, -x^5 + 1/2, -x^6)$$
$$(x^1, x^2, x^3, x^4, x^5, x^6) \xrightarrow{f} (-x^1 + 1/2, -x^2, x^3 + 1/2, x^4, -x^5, -x^6)$$
$$(x^1, x^2, x^3, x^4, x^5, x^6) \xrightarrow{h} (-x^1, -x^2, -x^3 + 1/2, -x^4, x^5 + 1/2, x^6)$$

Nice properties:

•Flat $\Rightarrow$ **CFT description**

• No fixed points $\Rightarrow$ smooth CY space, no twisted states.

• SUSY spontaneously broken: at large volume $\mathcal{N}=4$ $\Rightarrow$ adiabatic argument (Vafa,Witten) for perturbative S-duality.

• Chiral adjoints are massive.

Orbifold action on the CP factors

$$\gamma_g = (I_{n_o}, I_{n_g}) \quad , \quad \gamma_f = \gamma_h = (I_{n_o}, -I_{n_g}) \ ,$$

Gauge group : $SO(n_o) \otimes SO(n_g)$.

There are chiral multiplets in $(\mathbf{n_o}, \mathbf{n_g})$

O9/D9 tadpole conditions : $n_o + n_g = 32$.

There are no O5-planes, due to the free-orbifold action.

Particular case : $SO(32)$ gauge group, pure $N = 1$ SYM theory.

Result:

- The contribution of the instantons wrapping the first torus is universal (gauge-group independent).
- The contributions of the instantons wrapping the other tori are gauge-group dependent.
- Certain heterotic duals have an orbifold action in the twisted sector which is symmetric or asymmetric depending on the rank of the gauge group (by modular

invariance) $\rightarrow$ different twisted sectors. S-duality $\rightarrow$ correct instantonic interpretation in type I, but different signs in the threshold corrections between the two cases.

Conclusions

• We computed E1 instanton corrections in the BSGP type I orbifold model :

holomorphic corrections $\rightarrow f$.

non-holomorphic corrections $\rightarrow K$.

We expect similar results for the superpotential.

Similar results for all $N = 2$ sectors of $N = 1$ type I orbifolds.

• When combined with suitable fluxes and/or E5 effects, they generate moduli stabilization and SUSY breaking, changes in the soft spectra.

• S-dual of stringy instantonic effects in type I could teach us more about the heterotic string dynamics.

• The instantonic corrections to f and K deserve more phenomenological studies.