Kinetic terms in warped compactifications

Gonzalo Torroba

Department of Physics, NHETC, Rutgers University

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Based on [M. Douglas, GT; arXiv:0805.3700] and

work in collaboration with G. Shiu, B. Underwood, J. Shelton

Abstract: We develop formalism for computing kinetic terms in string compactifications with warping. This is based on the Hamiltonian of GR. Physical fluctuations turn out to obey a harmonic-type gauge condition, but depending on the warp factor. As an application, we work out the kinetic term of the complex modulus in the warped deformed conifold.

The aim is to understand better the sugra limit of string theory.

Requiring 4d maximal symmetry (AdS, Mink, dS), the most general background is a warped product,

$$ds^2 = e^{2A(y;\,u)}\, g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y;\,u)dy^i dy^j$$

where e^{2A} is the warp factor, and the internal metric g_{ij} depends on some parameters u^I.

How are warp effects encoded in the low energy dynamics?

This turns out to be very hard to understand and there is still a lot of work in progress in this direction.

Warp effects in EFT

The warp factor arises from backreaction of branes/fluxes on geometry. Some examples:

- AdS/CFT
- exponential hierarchies and low scale susy breaking
- dualities with confining gauge theories. (Kähler potential?)

An important question common to all these examples is what is the 4d EFT description for the previous metric fluctuations u^I.

It turns out that the effects of warping are encoded mainly in the kinetic terms. To see this, we will consider an example.

$\mathcal{N} = 1$ case

Consider type IIb with BPS fluxes and branes [DRS; GKP]. This preserves $\mathcal{N} = 1$ in 4d.

- The internal manifold is conformally equivalent to a CY, with the conformal and warp factors related:

$$ds^2_{10} = e^{2A(y;u)}\, \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y;\,u)}\, \tilde{g}_{ij}(y;\; u)\, dy^i dy^j$$

$\tilde{g}_{ij}$ is the CY metric and $\{u^I\}$ represents both complex and Kähler moduli.

Then the effective field theory for $\{u^I\}$ is described by the usual sugra expression

$$V = e^K \left(G^{I\bar{J}}\, D_I W\, D_{\bar{J}}\, W^* - 3|W|^2 \right)$$

- $W = W_{GVW}$ is not affected by warping
- so, do warp effects come from e^K or $G^{I\bar{J}}$?

⇝ Conjecture by [DeWolfe-Giddings]: warp corrections in sugra given by

$$K = -\log\left(\int e^{-4A}\,\Omega\wedge\bar{\Omega}\right) \Rightarrow G_{\alpha\bar{\beta}} = -\frac{1}{V_W}\int e^{-4A}\,\chi_\alpha\wedge\bar{\chi}_\beta$$

This is suggested by the fact that

$$V_{CY} = \int d^6y\,\sqrt{\tilde{g}_6} \rightarrow V_W = \int d^6y\sqrt{\tilde{g}_6}\,e^{-4A(y)}$$

To understand better this proposal, let's look at the warped deformed conifold [Klebanov-Strassler]

Example: deformed conifold

The complex modulus S parametrizes the size of the deformed 3-cycle, through which there are N units of F_3 flux.

[Douglas, Shelton, GT] computed the warp corrections to the Kähler metric:

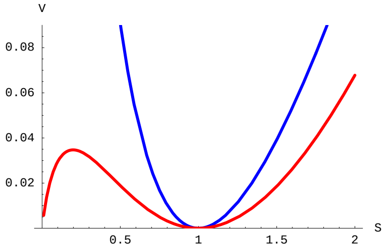
$$G_{S\bar{S}} = -\partial_S \partial_{\bar{S}} K = \frac{1}{V_W}\left[c \log \frac{\Lambda_0^3}{|S|} + c' \frac{(g_s N \alpha')^2}{|S|^{4/3}}\right]$$

- the log piece is the (unwarped) $\mathcal{N}=2$ contribution
- the warp factor introduces a new type of $|S|^{-4/3}$ divergence

Near the conifold point the new term dominates, producing large changes in the EFT:

$$V \propto |S|^{4/3} |D_S W|^2$$

Furthermore, in a model that breaks susy at small enough S, $G^{S\bar{S}}$ will produce a parametrically small scale of susy breaking.



However...

This proposal suffers from some problems,

- kinematics: the conjectured Kähler metric

$$G_{\alpha\bar{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha \wedge \bar{\chi}_\beta$$

is not diff invariant ($\chi \to \chi + d\lambda$). See also [Giddings and Maharana; Douglas, Shiu, GT, Underwood]

- dynamics: new light KK modes ... [Douglas, Shiu, GT, Underwood]

In any case, the upshot is that, quite generally, one expects kinetic terms to contain the main effect of warping.

Therefore we need to develop a method for computing kinetic terms in general warped backgrounds.

Formulating the problem

Start from a general warped solution which depends on certain parameters u^I,

$$ds^2 = e^{2A(y;\,u)}\, g_{\mu\nu}(x) dx^\mu dx^\nu + g_{ij}(y;\, u) dy^i dy^j$$

The standard procedure to compute 4d kinetic terms is to promote $u^I \to u^I(x)$ and extract

$$\int R_{10} \to \int d^4x \sqrt{g_4}\, g^{\mu\nu}\, G_{IJ}(u)\, \partial_\mu u^I \partial_\nu u^J$$

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[Giddings, Maharana] emphasized that this is not consistent, because $g_{MN}\big(y; u(x)\big)$ doesn't solve the 10d eoms. This turns out to be equivalent to the failure of G_{IJ} to be diff invariant.

Extra terms (proportional to derivatives ∂u …) are needed, to *compensate* for the time-dependence of $u(x)$.

Compensators in Yang-Mills theory

To understand the effect of compensating fields, consider a $U(1)$ gauge field

$$S = -\frac{1}{4}\int d^{10}x\,\sqrt{g_{10}}\,F^{MN}\,F_{MN}$$

and a family of solutions to $D^i F_{ij} = 0$ parametrized by u^I,

$$A_M = \big(A_\mu = 0,\ A_i(y;u)\big)$$

Substituting $u^I \to u^I(x)$, the kinetic terms give the metric

$$G_{IJ} = \int d^6y\sqrt{g_6}\,g^{ij}\,\frac{\partial A_i}{\partial u^I}\,\frac{\partial A_j}{\partial u^J}$$

However, this expression is not invariant under $\delta A_i = \partial_i\epsilon$. Since the original 10d action is invariant, there should be an error in the dimensional reduction.

The error is in assuming that $A_\mu = 0$ still holds for time-dependent moduli:

$$D^M F_{M\mu} = 0 \;\Rightarrow\; \partial_\mu \partial^i A_i = \partial^i \partial_i A_\mu \;\text{ cannot be solved by }\; \partial_\mu A_i \neq 0\,,\; A_\mu = 0$$

The new time-dependence forces a nonzero 4d component

$$A_\mu = \Omega_I \partial_\mu u^I \,,\; \partial^i \partial_i \Omega_I = \partial^i \frac{\partial A_i}{\partial u^I}$$

This is the simplest example of a *compensating field*.

The only effect of the compensator is to shift

$$\frac{\partial A_i}{\partial u^I} \to \delta_I A_i := \frac{\partial A_i}{\partial u^I} - \partial_i \Omega_I \;\text{ so that }\; \partial^i(\delta_I A_i) = 0$$

The field space metric is simply

$$G_{IJ} = \int d^6 y \sqrt{g_6}\, g^{ij}\, \delta_I A_i\, \delta_J A_j$$

Hamiltonian approach

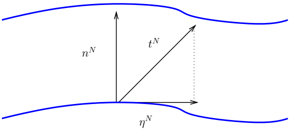
In analogy with the YM case, in warped compactifications time-dependent parameters will source off-diagonal components of the metric:

$$ds^2_{10} = e^{2A(y;u)}\, g_{\mu\nu}(x)\, dx^\mu dx^\nu + B^{(I)}_j(y)\, \partial_\mu u^I\, dx^\mu dy^j + g_{ij}(y;u)\, dy^i dy^j$$

However, the YM approach is hard to generalize to this case...

- √ It turns out that a direct way for finding the right gauge invariant kinetic terms is to derive the Hamiltonian of such warped backgrounds.

Review – Hamiltonian of GR



√ splitting g_{MN} : $\begin{cases} h_{MN} & \text{space-like piece} \\ \eta_N & \text{tangential shift} \end{cases}$

√ extrinsic curvature $K_{MN} = \frac{1}{2}(g^{tt})^{1/2}(\dot{h}_{MN} - \nabla_M \eta_N - \nabla_N \eta_M)$

√ canonical momentum $\pi_{MN} = \frac{\partial \mathcal{L}_G}{\partial \dot{h}_{MN}} = h^{1/2}(K_{MN} - h_{MN} K)$

√ $\mathcal{H}_G = \sqrt{-g_D}\left(-R^{(D-1)} + h^{-1}\pi^{MN}\pi_{MN} - \frac{1}{D-2}h^{-1}\pi^2\right) - 2\,\eta_N \nabla_M(\pi^{MN})$

√ η_N are Lagrange multipliers enforcing $\nabla_M(\pi^{MN}) = 0$

Kinetic terms

In our case, the time-dependence of h_{MN} is only implicit through $u^I(x)$. Computing the shift vectors,

$$\eta^i = B^i_I\, \dot{u}^I \implies \text{compensators = Lagrange multipliers of } \mathcal{H}_G \text{ !}$$

The dynamical variables of $\mathcal{H}_G$ define the following metric fluctuations:

$$K_{MN} \sim \dot{u}^I\, \delta_I h_{MN} := \dot{u}^I \frac{\partial h_{MN}}{\partial u^I} - \nabla_M \eta_N - \nabla_N \eta_M$$

$$\pi_{MN} \sim \dot{u}^I\, \delta_I \bar{h}_{MN} := \dot{u}^I \left(\delta_I h_{MN} - h_{MN}\, \delta_I h\right)$$

The only effect of compensating fields is to shift $\partial_I h_{MN} \to \delta_I h_{MN}$ (“physical” variation) and enforce the constraint

$$\nabla^M(\delta_I \bar{h}_{MN}) = 0$$

Finally, the kinetic term $\mathcal{H}_{kin}(\dot{u}, \dot{u}) = G_{IJ}(u)\, \dot{u}^I\, \dot{u}^J$ gives

$$G_{IJ}(u) = \int d^{D-1}x\, \sqrt{-g_D}\, g^{tt} \left(\delta_I \bar{h}_{MN}\, \delta_J \bar{h}^{MN} - \frac{1}{D-2}\, \delta_I \bar{h}\, \delta_J \bar{h} \right)$$

G_{IJ} defines a Riemannian metric on the space of metrics modulo gauge transformations. The constraint

$$\nabla^N (\delta_I \bar{h}_{NM}) = 0$$

implies that physical fluctuations are orthogonal to gauge transformations:

$$\mathcal{H}_{kin}(\nabla\, \epsilon,\, \delta h) = 0$$

Application to warped compactifications

Let us see how the previous formalism applies to general warped compactifications (possibly nonsupersymmetric):

$$ds^2 = e^{2A(y;\,u)}\, g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y;\,u)dy^i dy^j$$

The orthogonality constraints decompose into a 4d part and a 6d piece. The 4d part implies that the space-time canonical momentum vanishes, $\delta \bar{h}_{\mu\nu} = 0$. In terms of the physical fluctuations,

$$\delta_I\, e^{2A} = -\frac{1}{2}\, e^{2A}\, \delta_I g$$

This allows to eliminate the warp factor variation in terms of $\delta_I g$.

The 6d momentum may be written in terms of the internal metric fluctuations

$$\delta_I \bar{g}_{ij} = \delta_I g_{ij} + \frac{1}{d-2}\, g_{ij}\, \delta_I g$$

The internal part of the constraint implies that $\delta_I \bar{g}_{ij}$ is in harmonic gauge in the full warped metric; or, in terms of the 6d metric

$$\nabla^i (\delta_I \bar{g}_{ij}) + 3\, \partial^i A\, \delta_I \bar{g}_{ij} = 0 \qquad \text{(depends on the warp factor!)}$$

Then the general formula for the kinetic terms is

$$G_{IJ}(u) = \frac{1}{4} \int d^6 y \sqrt{g_6}\, e^{2A} \left(\delta_I \bar{g}_{ij}\, \delta_J \bar{g}^{ij} - \frac{1}{8}\, \delta_I \bar{g}\, \delta_J \bar{g} \right)$$

This metric is gauge invariant $\surd$

$\mathcal{N} = 1$ case – conformal CY

We return to one of the original motivations: understanding the Kähler metric for conformal Calabi-Yau backgrounds (type IIb w/BPS fluxes)

$$ds^2_{10} = e^{2A(y;u)}\,\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y;\,u)}\,\tilde{g}_{ij}(y;\,u)\,dy^i dy^j$$

In terms of the unwarped fluctuations, the constraints become

- $\delta_I A = \frac{1}{8}\delta_I\tilde{g} \implies$ nonzero trace induced by the warp factor
- $\tilde{\nabla}^i(\delta_I\tilde{g}_{ij} - \frac{1}{2}\tilde{g}_{ij}\,\delta_I\tilde{g}) = 4\,\partial^i A\,\delta_I\tilde{g}_{ij} \implies$ “warped” harmonic gauge

The warped moduli space metric then reads

$$G_{IJ}(u) = \frac{1}{4\,V_W}\int d^6y\sqrt{\tilde{g}_6}\,e^{-4A}\,\tilde{g}^{ik}\tilde{g}^{jl}\,\delta_I\tilde{g}_{ij}\,\delta_J\tilde{g}_{kl}$$

(in agreement with [Douglas, Shiu, GT, Underwood])

Properties of the warped field space metric

- Metric fluctuations are orthogonal to gauge transformations with respect to this metric. The kinetic term is gauge-invariant.
- Since the warp factor enters explicitly in the inner product, the orthogonality condition includes the warp factor and hence differs from harmonic gauge.
- The expression differs from the conjectured form

$$G_{\alpha\bar{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha \wedge \bar{\chi}_\beta$$

  which is constructed in terms of harmonic forms χ_α of the underlying CY.
- The warp factor seems to source terms which mix complex and Kähler moduli!

Kähler metric in the deformed conifold

To understand the new expression, let's compute the field space metric for the complex modulus S in the deformed conifold.

Consider the strongly warped limit, described by [Klebanov-Strassler]:

$$\begin{aligned} ds^2_{10} &= \frac{|S|^{2/3}}{(g_s N\alpha')} I(\tau)^{-1/2}\, \eta_{\mu\nu} dx^\mu dx^\nu + (g_s N\alpha')\, I(\tau)^{1/2} \Big\{ \frac{1}{3K(\tau)^2} (d\tau^2 + (g^5)^2) + \\ &+ K(\tau) \cosh^2\left(\frac{\tau}{2}\right) ((g^3)^2 + (g^4)^2) + K(\tau) \sinh^2\left(\frac{\tau}{2}\right) ((g^1)^2 + (g^2)^2) \Big\} \end{aligned}$$

$$\text{and warp factor } \; e^{-4A(\tau)} = \frac{(g_s N\alpha')^2}{|S|^{4/3}} I(\tau)\,.$$

In this regime, the 6d metric is independent of S, which only enters through the 4d redshift factor.

Instead, the warped metric fluctuation is determined completely by compensators,

$$\delta_S g_{ij} = -\nabla_i \eta_j - \nabla_j \eta_i$$

For this reason, the KS solution is very good for illustrating the effects of compensating fields.

Working in the hard-wall approximation, we solved the compensator equations and constructed $\delta_S g_{ij}$. The result is

$$G_{S\bar{S}} = \frac{k}{V_W} \frac{(g_s N \alpha')^2}{|S|^{4/3}}$$

which confirms the behavior found by [Douglas, Shelton, GT]. However, the precise numerical coefficient is different (smaller), because the correct projection orthogonal to gauge directions was used.

Comments on the Kähler potential

Based on the conifold results, we conclude with some comments on the structure of the Kähler metric.

The effect of compensating fields in the conifold turns out to be equivalent to a shift in the $(2,1)$ form by an exact piece

$$\chi_S \to \chi_S^{(w)} = \chi_S + d(b_{(2)}) \text{ so that}$$

$$d \star_6 (e^{-4A} \chi_S^{(w)}) = 0 \,,\; G_{S\bar{S}} = -\frac{1}{V_W} \int e^{-4A} \chi_S^{(w)} \wedge \star_6 \bar{\chi}_S^{(w)}$$

This expression for $G_{S\bar{S}}$ was derived for the conifold, although it may hold more generally (work in progress...)