

# Linear Sigma Models and $(0, 2)$ Mirror Symmetry

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Based on work in progress with Jock McOrist.

## Summary

- Mirror Symmetry is a powerful tool in the study of string vacua that preserve  $(2, 2)$  SUSY on the world-sheet.
- There are many interesting  $(0,2)$  Heterotic compactifications!
- *Is there Mirror Symmetry off the  $(2, 2)$  locus? Is it useful?*
- We wish to go beyond exactly soluble examples and describe moduli space, Yukawas, singularities, etc.
- We study **half-twisted** sectors of  $(0, 2)$  deformed linear sigma models for Calabi-Yau hypersurfaces in toric varieties and find encouraging clues:
  - **A/2** model is independent of “complex structure” moduli and solved by quantum restriction formula;
  - **B/2** model is independent of Kähler moduli and reduces to classical geometry.

## A Class of $(0, 2)$ Linear Sigma Models

- $d = 2$  gauge theory, gauge group  $G \simeq \text{U}(1)^{n-d} \times K$  with matter fields

$$\Phi_{(2,2)}^i = (\Phi^i, \Gamma^i), \quad i = 0, \dots, n$$

with charges  $Q_i^a$  under  $\text{U}(1)^{n-d}$ , and  $n - d$  neutral multiplets  $\Sigma_a$ .

- Chirality constraints:

$$\overline{\mathcal{D}}\Phi^i = 0 \quad \overline{\mathcal{D}}\Sigma_a = 0, \quad \overline{\mathcal{D}}\Gamma^i = E^i(\Phi, \Sigma).$$

- Action:  $S = S_{\text{kin}} + \left\{ \int d^2z d\theta^+ \mathcal{L} + \text{h.c.} \right\}$ ,

$$\mathcal{L} = \sum_{a=1}^{n-d} \frac{1}{8\pi i} \Upsilon_a \log q_a + \Gamma^0 P(\Phi^1, \dots, \Phi^n) + \Phi^0 \sum_{i=1}^n \Gamma^i J_i(\Phi^1, \dots, \Phi^n).$$

- $(0,2)$  SUSY constraint:  $E^0 P + \sum_{i>0} E^i J_i = 0$ .

## A Little Nomenclature

$$\mathcal{L} = \sum_{a=1}^{n-d} \frac{1}{8\pi i} \Upsilon_a \log q_a + \Gamma^0 P(\Phi^1, \dots, \Phi^n) + \Phi^0 \sum_{i=1}^n \Gamma^i J_i(\Phi^1, \dots, \Phi^n).$$

- **The V-model:** Drop  $(\Phi^0, \Gamma^0)$ . Can choose  $Q_i^a$  so that for generic  $q_a, E^i$  at low energies this is a (0,2) NLSM with target-space  $V$ —a projective toric variety of dimension  $d$  and left-moving bundle  $\mathcal{E} \rightarrow V$ , a deformation of  $T_V$ .
- **M-model:** Reinstate  $(\Phi^0, \Gamma^0)$ . When  $\mathcal{L}$  preserves  $U(1)_L \times U(1)_R$  symmetry and

$$Q_0^a = - \sum_{i>0} Q_i^a,$$

**M-model** believed to flow to a (0,2) fixed point.

## The (2,2) Locus

- (2, 2) SUSY  $\Leftrightarrow E^i = Q_a^i \sum_a \Phi^i$  for all  $i$ , and  $J_i = \frac{\partial P}{\partial \Phi^i}$  for  $i > 0$ .
- (2,2) Superpotentials:

$$\int d\theta^+ \mathcal{L} \rightarrow \int d\theta^+ d\bar{\theta}^- \widetilde{W}(q_a) + \int d\theta^+ d\theta^- \Phi^0 P(\Phi^i)$$

- May choose  $q_a$  so that at low energies **M-model** reduces to (2,2) NLSM with target-space Calabi-Yau hypersurface  $M \simeq \{P = 0\} \subset V$ .
  - Parameters:  $q_a \rightarrow$  complexified Kähler (toric);
- $$\Phi^0 P = \sum_r c_r \mu_r, \quad \mu_r = \prod_i (\Phi^i)^{p_{ri}}, \quad c_r \rightarrow \text{complex structure (polynomial)}.$$

## (2, 2) SUSY and “Linear” Mirror Symmetry

- Topological essentials:  $\mathcal{Q}_T^2 = 0$  ;  $S = S_{\text{top}} + \{\mathcal{Q}_T, V\}$ .

| Twist    | $\mathcal{Q}_T$                                       | Observables                  | Correlators   | computation        |
|----------|---|------------------------------|---|--------------------|
| <b>A</b> | $\overline{\mathcal{Q}}_+ + \overline{\mathcal{Q}}_-$ | $\mathcal{Q}_T \sigma_a = 0$ | $\langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle_M(q_a)$ | instanton sum      |
| <b>B</b> | $\overline{\mathcal{Q}}_+ + \overline{\mathcal{Q}}_-$ | $\mathcal{Q}_T \mu_r = 0$    | $\langle \mu_{r_1} \cdots \mu_{r_{d-1}} \rangle_M(c_r)$       | classical geometry |

- Quantum Restriction:

$$\langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle_M(q_a) = \langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \frac{-Q_0^a \sigma_a}{1 - Q_0^a \sigma_a} \rangle_V.$$

- **V-model** may be solved by toric methods.
- Compare **M-model** to **W-model**, W is Batyrev mirror of M:

$$\langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle_M(q_a) \leftrightarrow \langle \widehat{\mu}_{i_1} \cdots \widehat{\mu}_{i_{d-1}} \rangle_W(\widehat{c}_i),$$

$$q_a = \prod_i \widehat{c}_i^{Q_i^a} \text{ is a global mirror map.}$$

## (0,2) Deformations and the Half-Twists

- Take  $E^i = \sum_{a,j} \Sigma_a A_j^{ai} \Phi^j$ . Gauge invariance  $\implies$

$A^a = \text{diag}(A_{(1)}^a, \dots, A_{(J)}^a), A_{(\alpha)}^a$  are  $n_\alpha \times n_\alpha$  matrices of parameters.

- Deform  $J_i = \frac{\partial P}{\partial \Phi^i}$  by parameters  $\gamma$  while satisfying  $E^0 P + \sum_i \Phi^i J_i(\gamma) = 0$ .
- Low Energy: (0,2) NLSM with target  $M$  and bundle  $\mathcal{F}(A, \gamma) \rightarrow M$ .  $\mathcal{F}(0, 0) = T_M$ .
- Two distinct half-twists (**A/2, B/2**). Both have  $\mathcal{Q}_T = \overline{\mathcal{Q}}_+$ .
- Observables:  $\mathcal{Q}_T \sigma_a = 0$  in **A/2**,  $\mathcal{Q}_T \mu_r = 0$  in **B/2**.
- $\mathcal{Q}_T^2 = 0 \implies$ 

$$\begin{aligned} \langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle_M &= F(q, A, c, \gamma), \\ \langle \mu_{r_1} \cdots \mu_{r_{d-1}} \rangle_M &= G(q, A, c, \gamma). \end{aligned}$$

Is there further decoupling?

## Localization and Zero Modes

- Fixed point theorem: half-twisted path integral localizes onto field configurations annihilated by  $Q_T$ .
- In either half-twist, this locus is given by gauge instantons:

$$\left. \begin{aligned} D_a + f_a &= 0 \\ \phi^0 &= 0 \\ D_{\bar{z}}\phi^i &= 0 \\ P &= 0 \end{aligned} \right\} \rightarrow \mathcal{M}_n \text{--- V-model instanton moduli space}$$

$$\rightarrow \text{a complicated subset of } \mathcal{M}_n$$

- Expand the action around a point in  $\mathcal{M}_n$  and examine the fermion zero modes. Use holomorphy to show

$$\langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle_M = \langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \frac{-A_0^a \sigma_a}{1 - A_0^a \sigma_a} \rangle_V \quad \text{quantum restriction}$$

$$\langle \mu_{r_1} \cdots \mu_{r_{d-1}} \rangle_M = G(c, A, \gamma) \quad \text{classical geometry.}$$

## An Example of Quantum Restriction

- **V-model:**  $V$  is resolved  $\mathbb{P}^4_{1,1,2,2,2}$ .  $Q_i^a = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & -2 \end{pmatrix}$ .

Parameters:  $q_1, q_2, \overline{\mathcal{D}} \begin{pmatrix} \Gamma^1 \\ \Gamma^2 \end{pmatrix} = \begin{pmatrix} \Sigma_2 + \epsilon_1 \Sigma_1 & \epsilon_2 \Sigma_1 \\ \epsilon_3 \Sigma_1 & \Sigma_2 \end{pmatrix} \begin{pmatrix} \Phi^1 \\ \Phi^2 \end{pmatrix}$ .

- **M-model:** a degree  $\binom{4}{0}$  hypersurface in  $V$ , *any*  $J_i$ .
- **Quantum Restriction:**

$$\langle \sigma_1^4 \rangle = 2 [(1 - 2^8 q_1)^2 - 2^{18} q_1^2 q_2 + 2\epsilon_1(1 - 2^8 q_1) - 4\epsilon_2\epsilon_3]^{-1}.$$
- **Parameter democracy!**
- **Singularity signals opening up of the “Coulomb” branch—a massless  $\sigma$  direction in field space predicted by the one-loop effective potential**

$$\mathcal{L}_{\text{eff}} = \Upsilon_a J_a(\Sigma; q, A)$$

obtained by integrating out massive  $(\Phi^i, \Gamma^i)$  matter.

## Conclusions, Comments, and Further Directions

- Studied  $(0,2)$  deformations of  $(2,2)$  linear sigma models:  
A/2 correlators in **M**-model obtained by quantum restriction from **V**-model.  
B/2 correlators are given by classical computations.
- Correlators of **M** and **W** models will help us determine a “linear”  $(0,2)$  mirror map.
- Counting parameters in linear model is straight-forward and passes a number of checks.
- **V**-model comments:
  - Computations are made tractable by effective potential techniques.
  - Deformed quantum cohomology is easily determined.
- Likely that **B/2** correlators are  $A$ -independent. Is this true?
- Is there a “linear”  $(0,2)$  mirror map?
- What is the  $(0,2)$  version of special geometry?
- Can our techniques be applied to more generic  $(0,2)$  theories?