Conformal collider physics: Energy correlation functions

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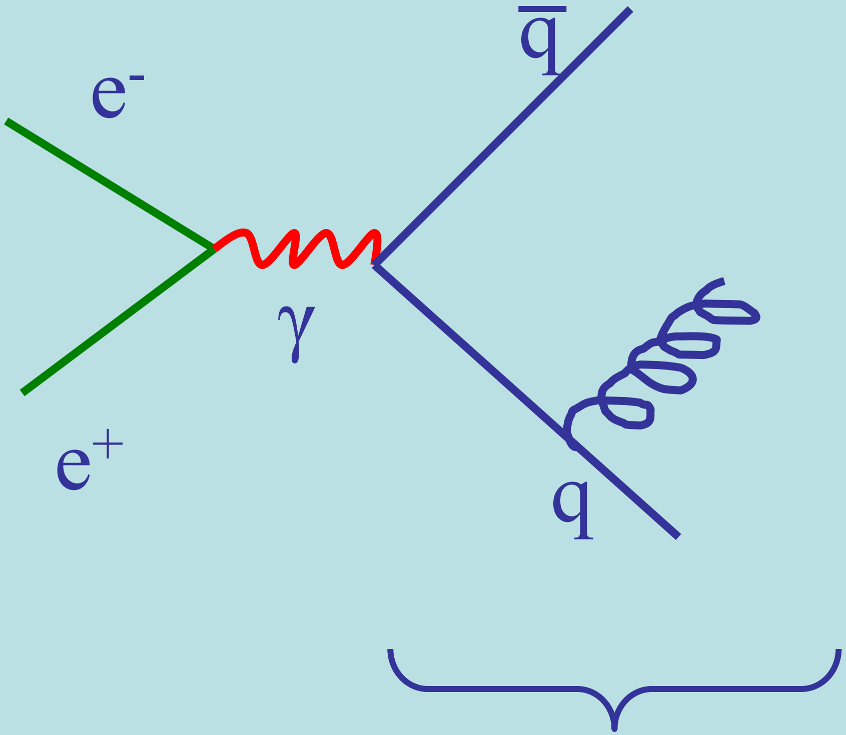
Conformal collider physics

Produce a localized excitation in a CFT



$J^{1}\,(p)j0i$

Understand how it decays.

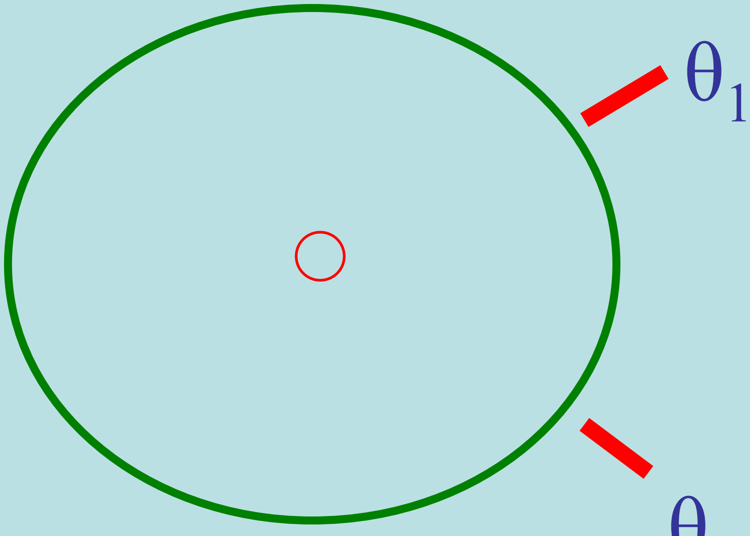
How do we describe the final state ?

Motivation

- QCD → conformal is simpler
- Beyond the Standard Model Physics
- Other conformal theories we might encounter in nature
- Understand this at strong coupling via the AdS dual.

How do we describe the produced state ?

- Not convenient to talk about partons
- Inclusive observable (jet cross sections) Sterman, Weinberg 77
- Energy correlation functions. Basham, Brown, Ellis, Love 78



$$\langle E(\theta_1)\rangle$$

$$\langle E(\theta_1)E(\theta_2)\rangle$$

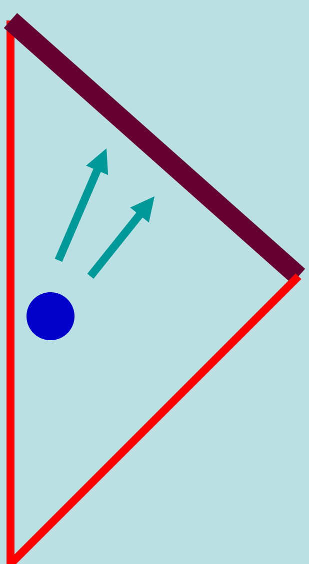
$$\langle E(\theta_1)\cdots E(\theta_n)\rangle$$

Integrated flux of energy at infinity

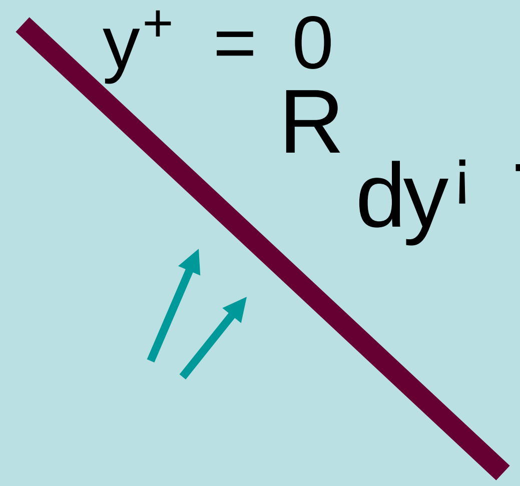
$$\mathcal{E}(\theta) = \lim_{r\to\infty} r^2 \int dt\, T^{0i} n^i$$

$$\langle \mathcal{E}(\theta) \rangle = \frac{\langle 0|\psi^\dagger \int T\, |\psi|0\rangle}{\langle 0|\psi^\dagger\, |\psi|0\rangle}$$

Three point function



conformal transformation



One point functions

$$h^2(\mu)i \ = \ \frac{h0jj^{y} \ \text{R} \ T \ j \ j0i}{h0jj^{y} \ j \ j0i}$$

$$h^2(\mu)i \ = \ \frac{E}{4\text{¼}} \ \text{i} \ 1 + a_2(\cos^2\mu \ \text{¡} \ \tfrac{1}{3}) \ \text{¢}$$

(in CM frame)

One undetermined constant in the j j T three point function.

In an N=1 SUSY theory

Take j = R-current

$$a_2 \ = \ 3(c \ \text{¡} \ a){=}c$$

$$T^1_1 \ \text{»} \ cW^2 \ \text{¡} \ a(\text{Euler})$$

Can we get a non-zero a_2 from AdS ?

Bulk couplings between an on shell graviton and two on shell photons

$$S_{5-d} \approx \int F^2 + a_2 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

-Only two possible couplings in 5 dimensions
-First coupling determined in terms of the jj two point function
- The second coupling is a higher derivative correction in the bulk

Repeat with a state created by the stress tensor.

2 coefficients in a general theory (+ one fixed from the two point function)

Related to bulk higher derivative terms

Only one coefficient in an N=1 SCFT

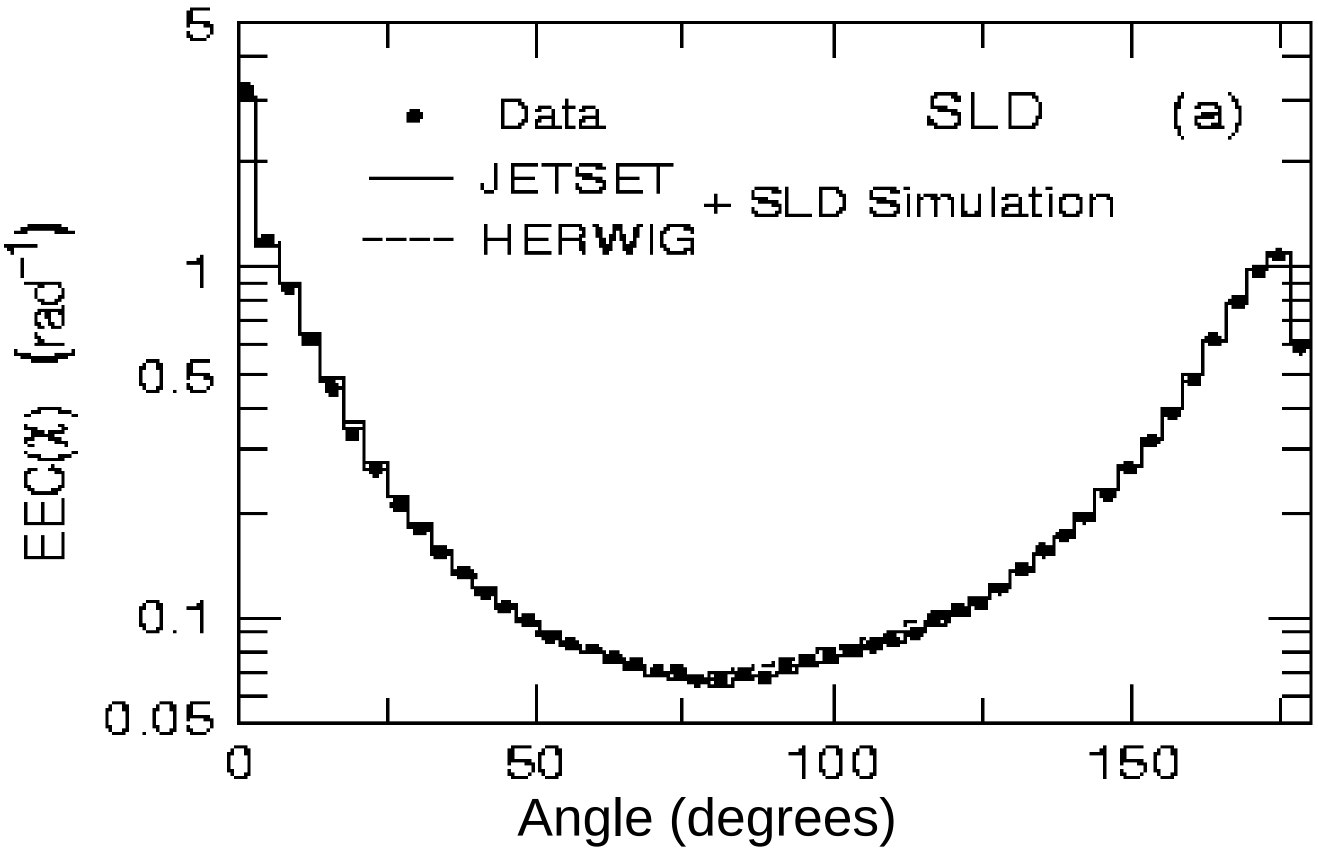
Positivity → $\frac{3c}{2} \gtrless a \gtrless \frac{c}{2}$

Two point function

Two point function is a function of the angle between the two detectors

Basham, Brown, Ellis, Love 78
+ …..

Energy correlations SLD collaboration

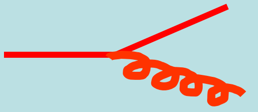


arXiv:hep-ex/9405006

Small angle behavior

very weak coupling:

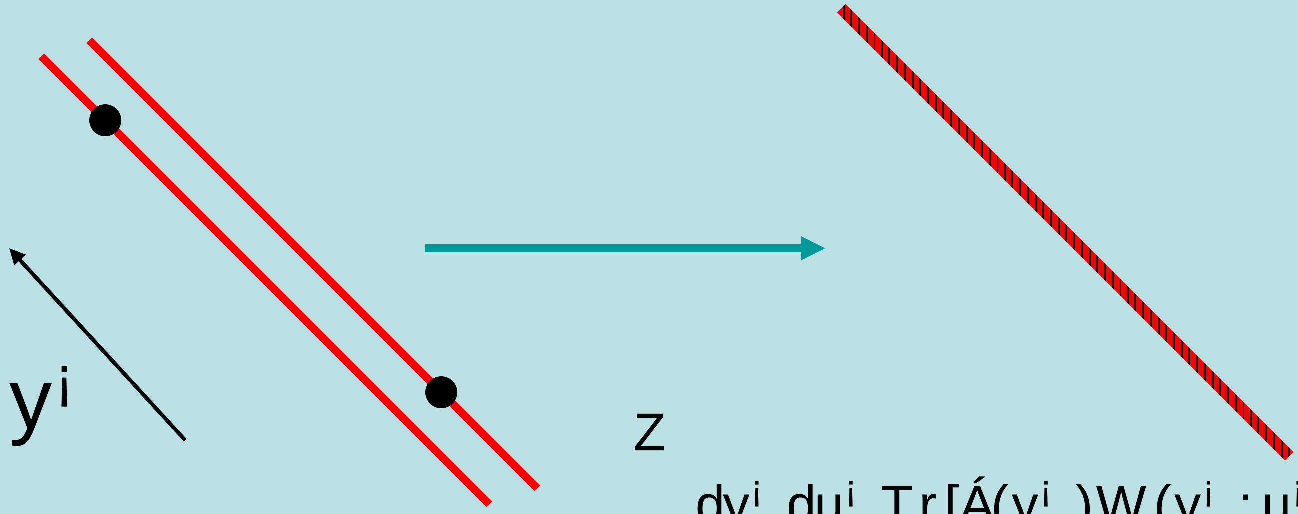
$h^2(\mu_1)^2(\mu_2)i$ » $g^2 N \frac{1}{\mu_{12}^2}$; μ_{12} ¿ 1



Small angle behavior

$$\langle {}^2(\theta_1)\,{}^2(\theta_2)\cdots\rangle \approx \sum_n |\theta_{12}|^{t_n - 4}\langle U_n(\theta_2)\cdots\rangle$$

U is a non-local light-ray operator of spin $j = 3$. (spin = boost in [+,-] directions). t_n is its twist.



$$dy^- \, du^- \, \mathrm{Tr}[\phi(y^-)W(y^-;u^-)\phi(u^-)](y^- - u^-)^{-4}$$

Twist = Δ - S

$^2(y)^2(0) =$

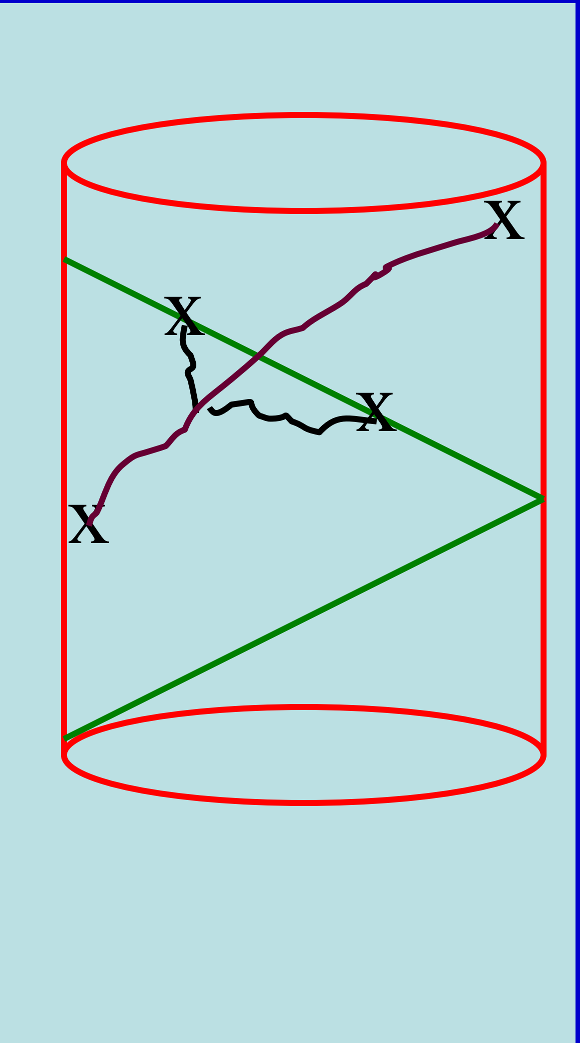
$= \int dy^{i}\, T_{i\;i}\,(y) \int dy^{i}\, T_{i\;i}\,(0)$ » $y^{i\;4+¿n} \int dy^{i}\, O^{n}_{i\;i\;i}$

Non-local operator of spin 3

Balitsky Braun 89

Konishi, Ukawa, Veneziano 79

Strong coupling computation

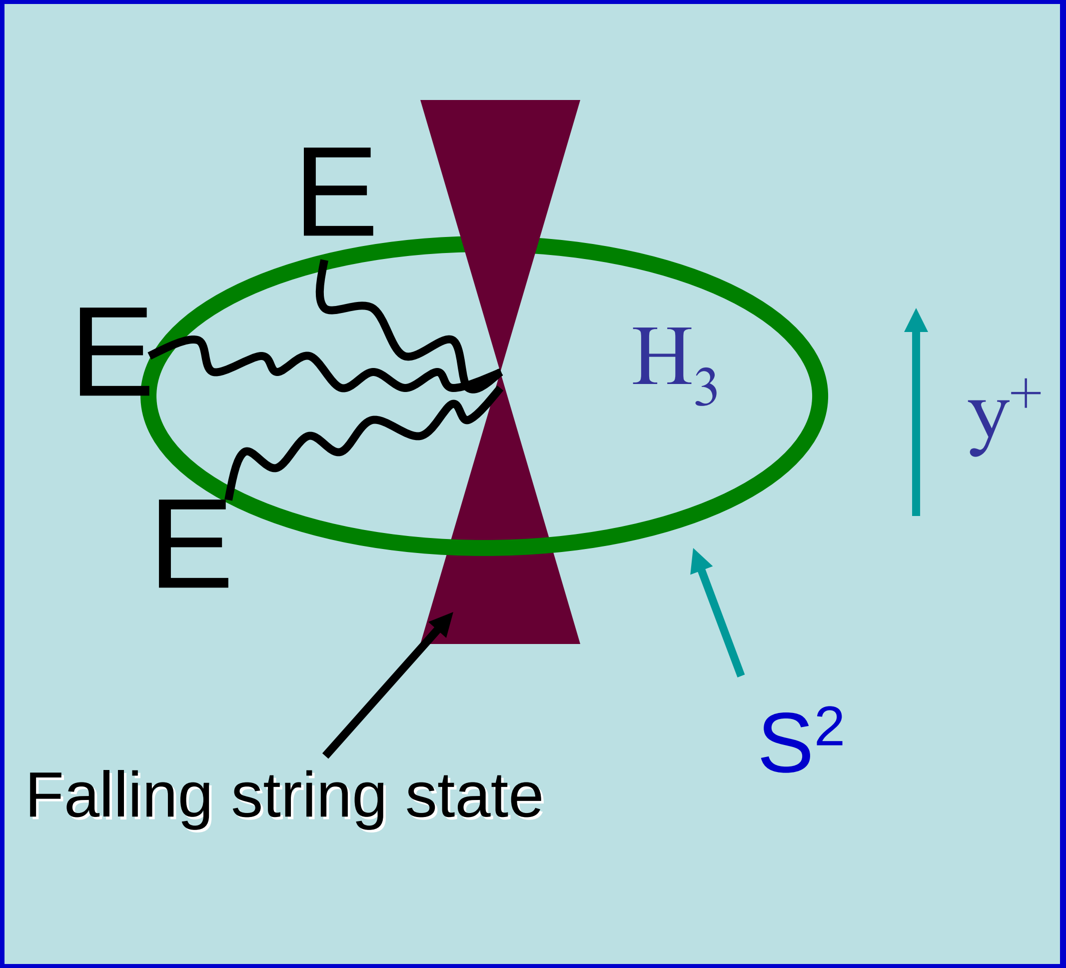


Since we integrate the stress tensor → bulk graviton: shock wave localized on the horizon.

R x H_3 subspace (SO(1,3) symmetry)

Energy correlations → taking a ``snapshot'' as the falling string crosses the AdS horizon.

Snapshot crossing the horizon



Results in gravity

Energy is uniform with no fluctuations.

$$\langle E(\theta_1)\cdots E(\theta_n)\rangle = \left(\frac{E}{4\pi}\right)^n$$

Very copious gluon emission,
rapid fragmentation at strong coupling

Polchinski
Strassler

Results in string theory

We have small fluctuations.

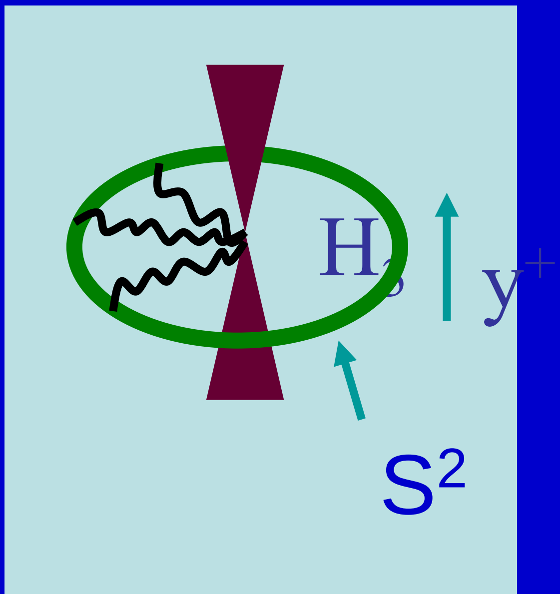
$$\langle E(\theta_1)E(\theta_2)\rangle = \left(\frac{E}{4\pi}\right)^2 \left[1+\frac{1}{\lambda}\cos^2\theta_{12}+\cdots\right]$$

Computed using flat space and then transferring the result to AdS.

There is also a non-trivial three point function.

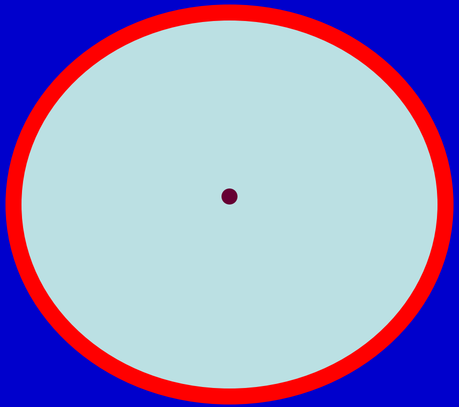
Fluctuations are not gaussian.

Locality in AdS and energy distributions

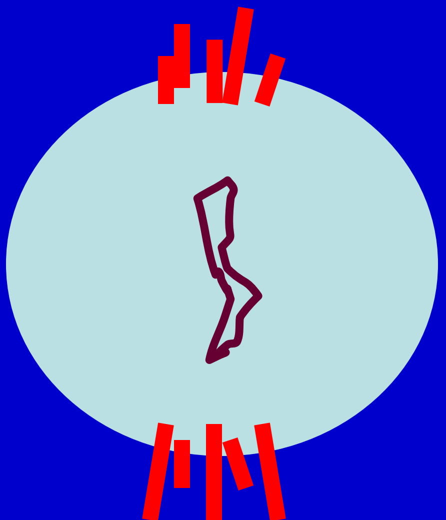


The center of mass is at a definite point on H^3

This point is given by the direction of the 4-momentum of the inserted operator

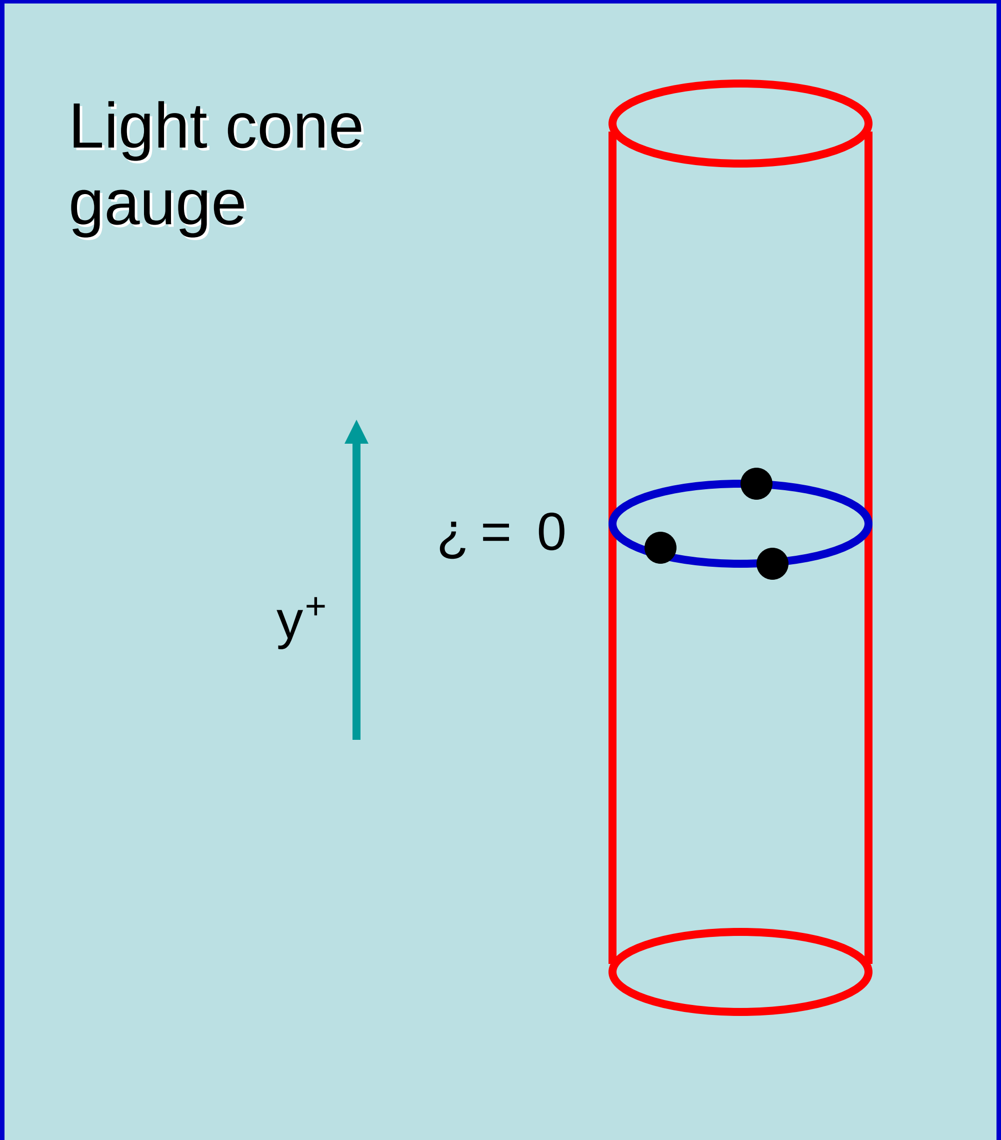


Localized in AdS = Completely uniform on the boundary



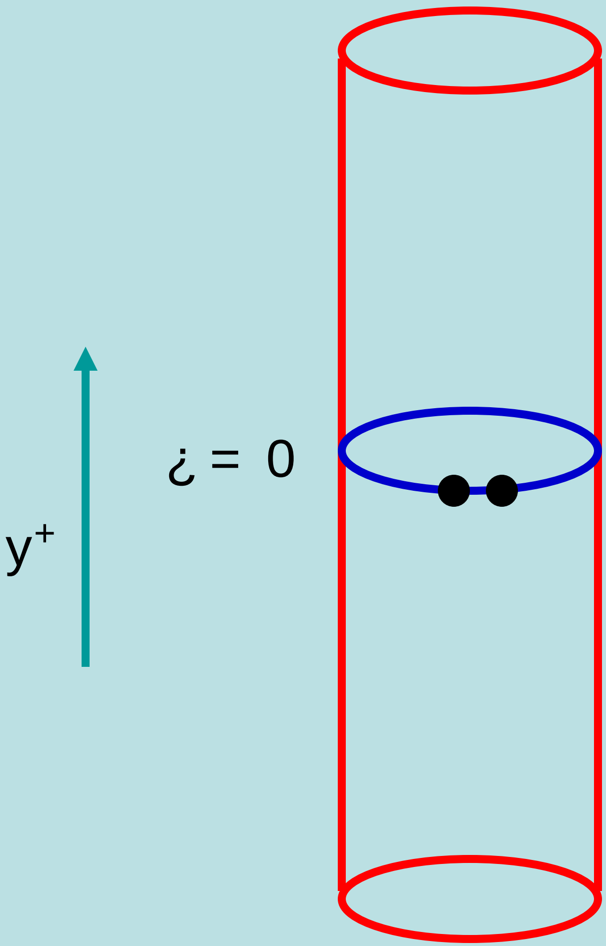
Spread in AdS = More localized on the boundary

The string computation



gravitational shock wave → localized in y^+ → localized at one particular worldsheet time.

OPE in string theory



Worldsheet OPE →

Spacetime OPE

Unusual worlsheet operators→ non-local light ray operators

Twist = $2^{1=2}$ ¸ $^{1=4}$

$$\tau_3 = 2 + c_1 \lambda + \cdots \qquad \text{for} \quad \lambda \ll 1$$

$$\tau_3 = 2^{1/2} \lambda^{1/4}, \qquad \text{for} \quad \lambda \gg 1$$

Kotikov
Lipatov

$$\langle \varepsilon^2(\theta_1)\varepsilon^2(\theta_2)\cdots\rangle \sim \sum_n |\theta_{12}|^{t_n - 4} \langle U_n(\theta_2)\cdots\rangle$$

Localized jets → Twist ~ 2

More uniform distribution → Large twist

Conclusions

- Studied energy correlations in conformal field theories
- Small angle behavior is characterized by non-local light-ray operators with definite spin
- One point functions → three point functions in the CFT → Few undetermined coefficients → higher derivative corrections in gravity. Bounds on the coefficients. Bounds on a and c.
- Gravity prescription: Probe the string as it crosses the horizon with gravitational shock waves.
- Gravity = completely uniform events
- Stringy corrections → small fluctuations. OPE
- Localized on the horizon → uniform on S^2

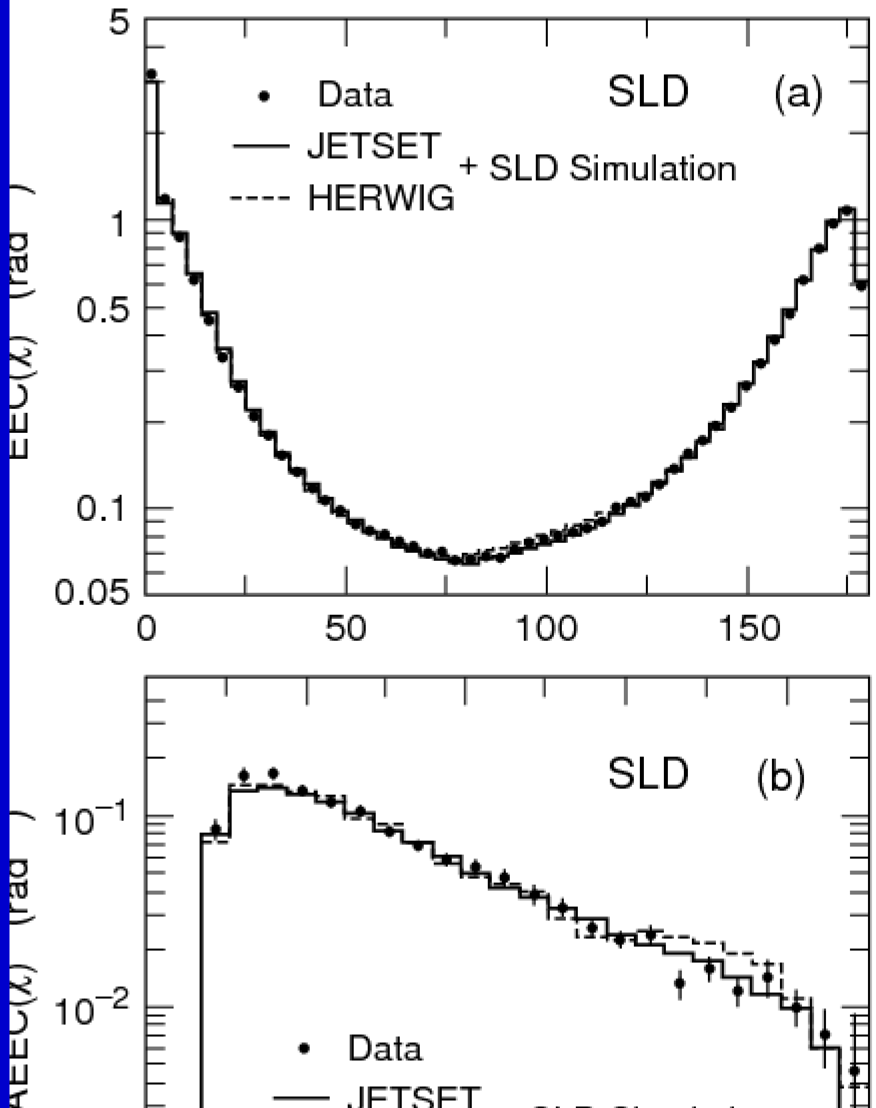
Future

- Non-conformal theories
- Small angle behavior in non-conformal theories
- Hadronization corrections (Can be large or small)
- Non-conformal cases with gravity duals
- More complicated initial states, (similar to protons)

Related to deep inelastic scattering

It is related to a particular moment of the deep inelastic amplitude of gravitons.

$$\langle \epsilon(-q)\epsilon(q)\rangle \sim \int_{-\infty}^{\infty} ds\, \sigma_{DIS}(s;q^2)$$



Data
JETSET
HERWIG
+ SLD Simulation
SLD
(a)
5
1
0.5
0.1
0.05
0
50
100
150
EEC(χ)
SLD
(b)
10⁻¹
10⁻²
AEEC(χ)
Data