String Phenomenology 2008

Calabi-Yau threefolds with abelian fibrations and $Z/2Z$ actions

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(based on a joint work with Ron Donagi)

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1. Motivation from $SU(5)$ heterotic standard model

From Heterotic string compactification, there is a way to obtain the Standard Model[1]. Mathematically, this leads to the following problem.

Problem: Find a smooth Calabi-Yau 3-fold Z with a Kähler form ω and a reductive subgroup $G' \subset E_8$ so that

(1) the centralizer G of G' in E_8 is a group isogenous to $SU(3) \times SU(2) \times U(1)$;

(2) there exsits an ω-stable $G'_{\mathbf{C}}$-bundle $\mathcal{V} \longrightarrow Z$ so that

- $c_1(\mathcal{V}) = 0$,
- $c_2(Z) - c_2(\mathcal{V})$ is the class of an effective reduced curve on Z, **(anomaly cancellation)**
- $c_3(\mathcal{V}) = 6$. **(3-generations condition)**

[1] Bouchard Talk in Wednesday

We focused on the case of $G' = SU(5) \times \mathbf{Z}/2\mathbf{Z}$. In this case, we have

$$G = \text{centralizer of } G' \text{ in } E_8 = SU(3) \times SU(2) \times U(1)$$

In order to obtain concrete examples, we look for Z with $\pi_1(Z) = \mathbf{Z}/2\mathbf{Z}$. Moreover $\mathcal{V}$ is $SL(5, \mathbf{C}) \times \mathbf{Z}/2\mathbf{Z}$-bundle on Z. Let

$$\pi : \tilde{Z} \longrightarrow Z$$

be the universal cover such that $Z = \tilde{Z}/ < \tau >$ where τ is the fixed point free automorphism of $\tilde{Z}$ of order 2. Such $\mathcal{V}$ can be obtained as $\mathcal{V} = \pi_* \pi^* V_0$ where V_0 is a $SL_5(\mathbf{C})$-bundle on Z. Note that $V = \pi^*(V_0)$ is the τ-invariant $SL_5(\mathbf{C})$-bundles on $\tilde{Z}$. From this considertation, one can have the following equivalent data:

$$(Z, \omega, \mathcal{V}) \Longleftrightarrow (\tilde{Z}, \tau, H, V)$$

Search for $\mathbf{Z}/2\mathbf{Z}$-examples:

Find

- $\tilde{Z}$: a smooth simply connected Calabi-Yau 3-fold with a fixed point free invlolution $\tau : \tilde{Z} \longrightarrow \tilde{Z}$ and an ample line bundle H (Kähler structure on $\tilde{Z}$),
- V: an H-stable holomorphic vector bundle of rank 5 on $\tilde{Z}$ such that
  - **(I)** V is τ-invariant,
  - **(C1)** $c_1(V) = 0$,
  - **(C2)** $c_2(\tilde{Z}) - c_2(V)$ is effective,
  - **(C3)** $c_2(V) = 12$. $\Leftrightarrow c_3(\mathcal{V}) = 6$.

The Only Known Example which gives $SU(5)$-heteroticMSSM

$(\tilde{Z}, \tau, H, V)$: (Bouchrad Talk on Wednesday, R. Donagi, B.A. Ovrut, T. Pantev, D. Waldram, V. Bouchard and R. Donagi)

- $\tilde{Z} = B \times_{\mathbf{P}^1} B'$, the fiber product of some rational elliptic surfaces $\beta : B \longrightarrow \mathbf{P}^1$ and $\beta' : B' \longrightarrow \mathbf{P}^1$ with a fixed point free involution $\tau : \tilde{Z} \longrightarrow \tilde{Z}$, and
- V: a stable bundle with respect to some ample line bundle H satisfying the conditions **(I)**, **(C-I, II, III)**.

$$\begin{array}{ccccc} & & \tilde{Z} & & \\ & \pi \swarrow & & \searrow \pi' & \\ B & & & & B' \\ & \beta \searrow & & \swarrow \beta' & \\ & & \mathbf{P}^1 & & \end{array}, \quad 0 \longrightarrow V_2 \longrightarrow V^* \longrightarrow V_3 \longrightarrow 0$$

Remarks

(1) Bouchard and Donagi calculated the various cohomology groups

$$H^q(\tilde{Z}, V^*)_\pm, H^q(\tilde{Z}, \wedge^2 V^*)_\pm$$

which gives the particle spectrum of the compactification.

(2) Bouchard, Cvetič and Donagi calculated the superpotenstial tri-linear coupluings of the example.

Search for new examples of $SU(5)$ heterotic standard model

(1) Bouchard and Donagi classified all smooth Calabi-Yau threefolds $\tilde{Z}$ and finite fixed point free automorphisms on them where $\tilde{Z}$ have the structure of fiber products of rational elliptic surfaces

(2) Mark Gross and S. Popescu constructed Calabi-Yau threeflod $\hat{V}$ with $\pi_1(\hat{V}) \simeq (\mathbf{Z}/8\mathbf{Z})^2$.It has the structure of abelian fibrations $\pi : \hat{V} \longrightarrow \mathbf{P}^1$ whose generic fibers have a polarization H of type (1, 8). (M. Gross's talk, Bak, Bouchard and Donagi constructed rank 5 bundle with the conditions above).

(3) L. Borisov and Z. Hua constructed Calabi-Yau threefolds with nonabelian fundamental groups.

Our examples (based on my construction in 1998))

(1) Our example is a smooth simply connected Calabi-Yau threefold J with a fibration $\varphi : J \longrightarrow \mathbf{P}^1$ of abelian surfaces with principal polarization. Moroever there exists a family of curves of genus 2 $f_1 : Y \longrightarrow \mathbf{P}^1$ with two sections s_0, s_∞ such that for each $t \in \mathbf{P}^1$, the fiber J_t is the Jacobian variety of the curve Y_t of genus 2. (That is $J_t = Pic^0(Y_t)$). Moreover we have a commutative diagram

$$\begin{array}{ccccc} Y & & \overset{\iota}{\hookrightarrow} & & J \\ f_1 & \searrow & & \swarrow & \varphi \\ & & \mathbf{P}^1 & & \end{array}$$

such that s_0 maps to the zero section of $J \longrightarrow \mathbf{P}^1$.

(2) The class $\sigma = s_\infty - s_0$ determines the translation automorphism $\tau_\sigma : J \longrightarrow J$ over $\mathbf{P}^1$ of order two.

$$\begin{array}{ccc} J & \xrightarrow{\tau_\sigma} & J \\ \varphi \searrow & & \swarrow \varphi \\ & \mathbf{P}^1 & \end{array}$$

τ_σ is a fixed point free involution on J.

The Hodge diamond of J $h^{1,1}(J) = h^{1,2}(J) = 14, c_3(J) = 0.$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 14 & & 0 & \\ 1 & & 14 & & 14 & & 1 \\ & 0 & & 14 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

The quotient $J' = J/ < \tau_\sigma >$

The quotient J' is a Calabi-Yau threefold with $\pi_1(J') \simeq \mathbf{Z}/2\mathbf{Z}$. It also has the fibration of abelain surface $\varphi' : J' \longrightarrow \mathbf{P}^1$.

The Hodge diamond of J'

$h^{1,1}(J') = h^{1,2}(J') = 10, c_3(J') = 0.$

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 10 & & 0 & \\
1 & & 10 & & 10 & & 1 \\
 & 0 & & 10 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Some Remark on J

(1) Generic complex deformation of Calabi-Yau 3-fold J has no fixed point free involution τ.

(2) Some part of A-model Yukawa coupling for genus 0 can be calculated by means of theta functions of Mordell-Weil lattice of $\varphi : J \longrightarrow \mathbf{P}^1$. The generic Mordell-Weil lattice is isomorphic to D_{12}^+. (See my paper in 2000).

(3) Further specialization of Y may give a Calabi-Yau 3-fold J with $(\mathbf{Z}/2\mathbf{Z})^3$-actions.

Construction of J = Construction of Y

(1) Take $\Sigma_0 = \mathbf{P}^1 \times \mathbf{P}^1$. Take any divisior B of type $(6, 2)$.
(2) Take the double cover $\pi : Y' \longrightarrow \mathbf{P}^1$ branche along B. Y is the minimal resolution of the singularities of Y'.

$$\begin{array}{ccc} Y & \longrightarrow & Y' \\ \downarrow & & \downarrow \\ \widetilde{\Sigma_0} & \longrightarrow & \mathbf{P}^1 \times \mathbf{P}^1 \end{array}$$

Then the projections $p_i : \mathbf{P}^1 \times \mathbf{P}^1 \longrightarrow \mathbf{P}^1, (i = 1, 2)$ induce the maps $f_i : Y \longrightarrow \mathbf{P}^1$, where $f_i = p_i \circ \pi$. Then f_1 induces a family of curves of genus 2 and f_2 induces a conic bundle structure

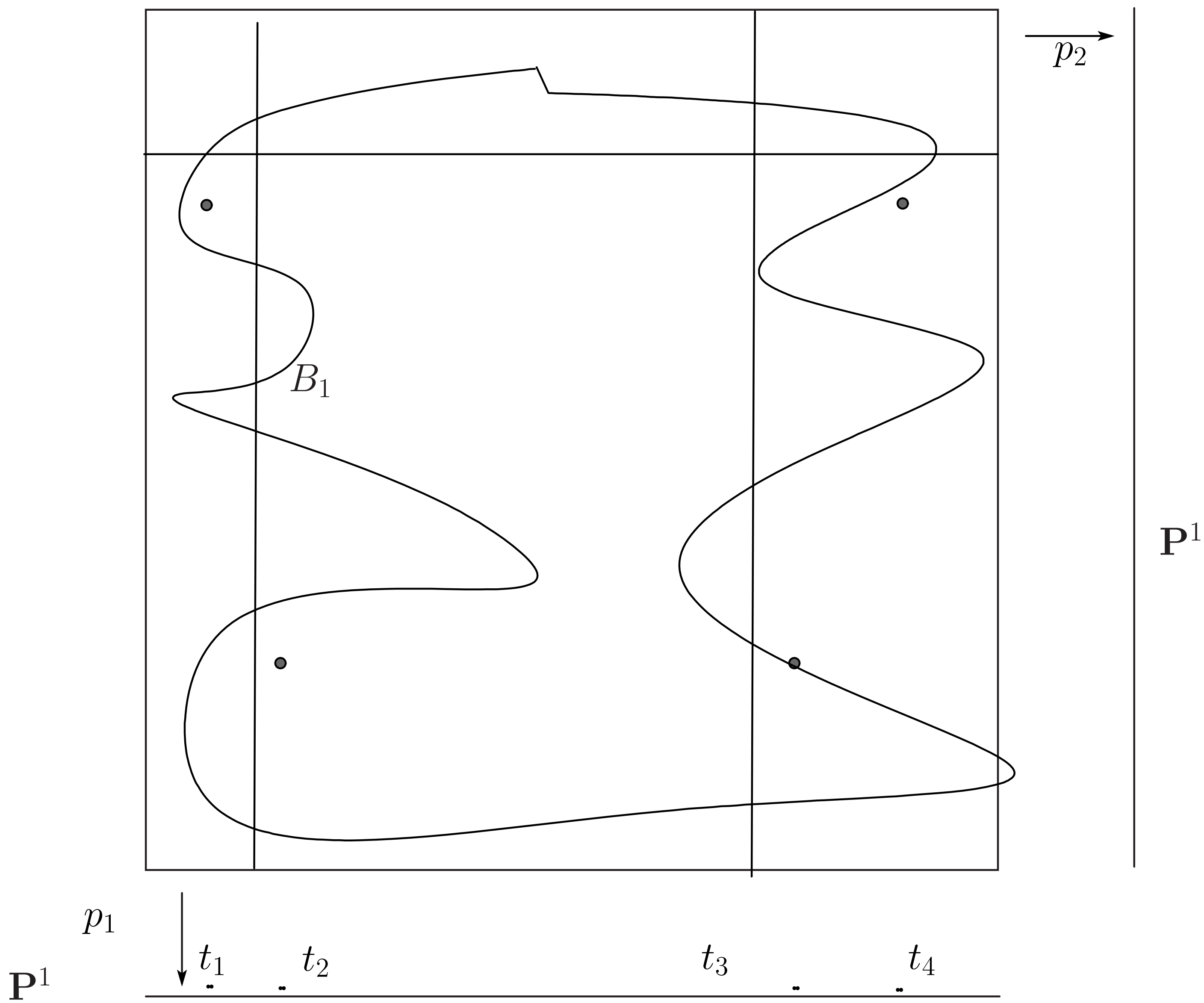
$$\begin{array}{ccc} & Y & \\ f_1 \swarrow & & \searrow f_2 \\ \mathbf{P}^1 & & \mathbf{P}^1 \end{array}$$

Further investitgations

(1) Construct rank 5-bundles V on J satisfying the conditions for a heterotic $SU(5)$ MSSM.
(2) Calculate the cohomology groups.
(3) Study the trilinear coupling for J.

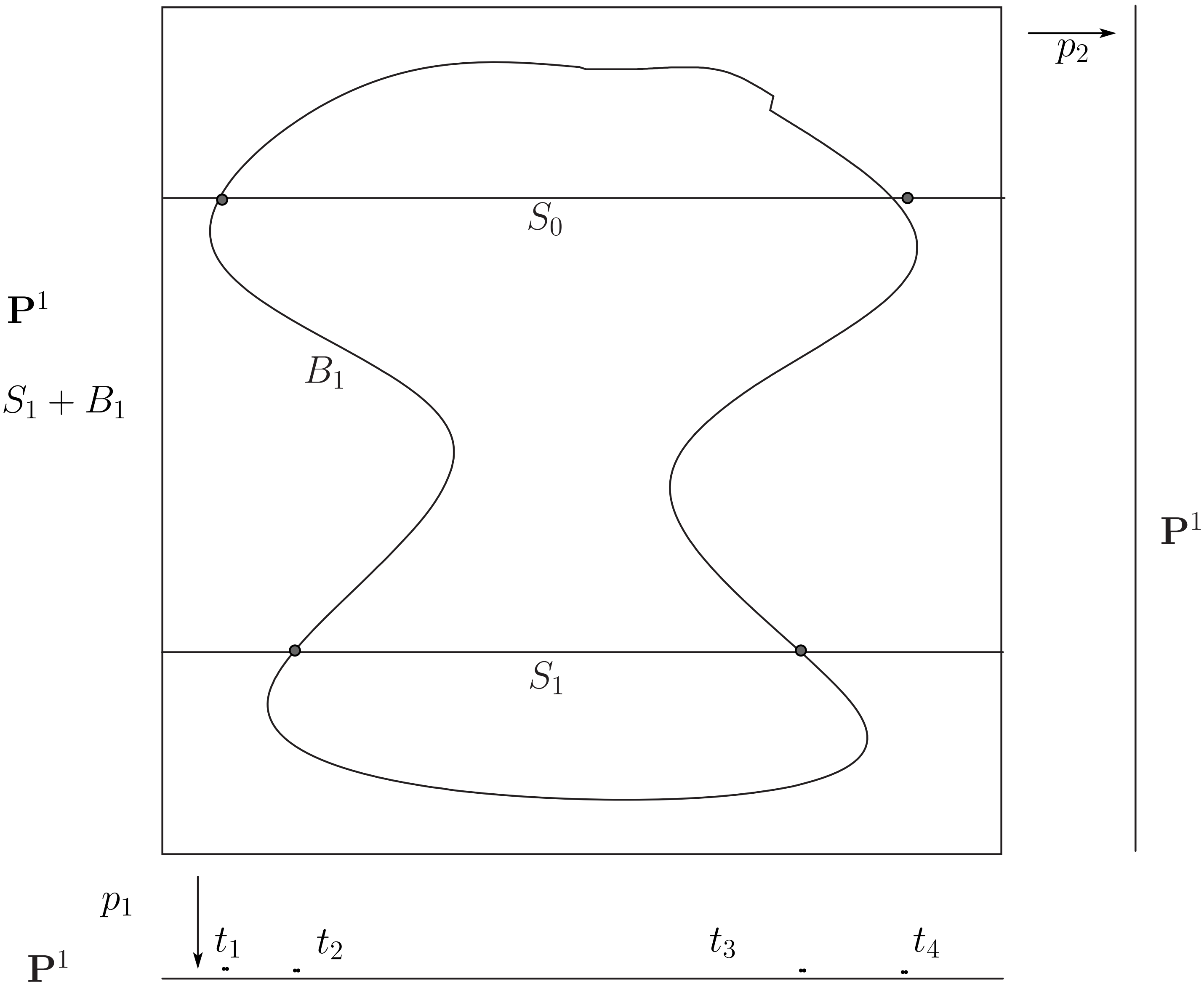
Generic cases

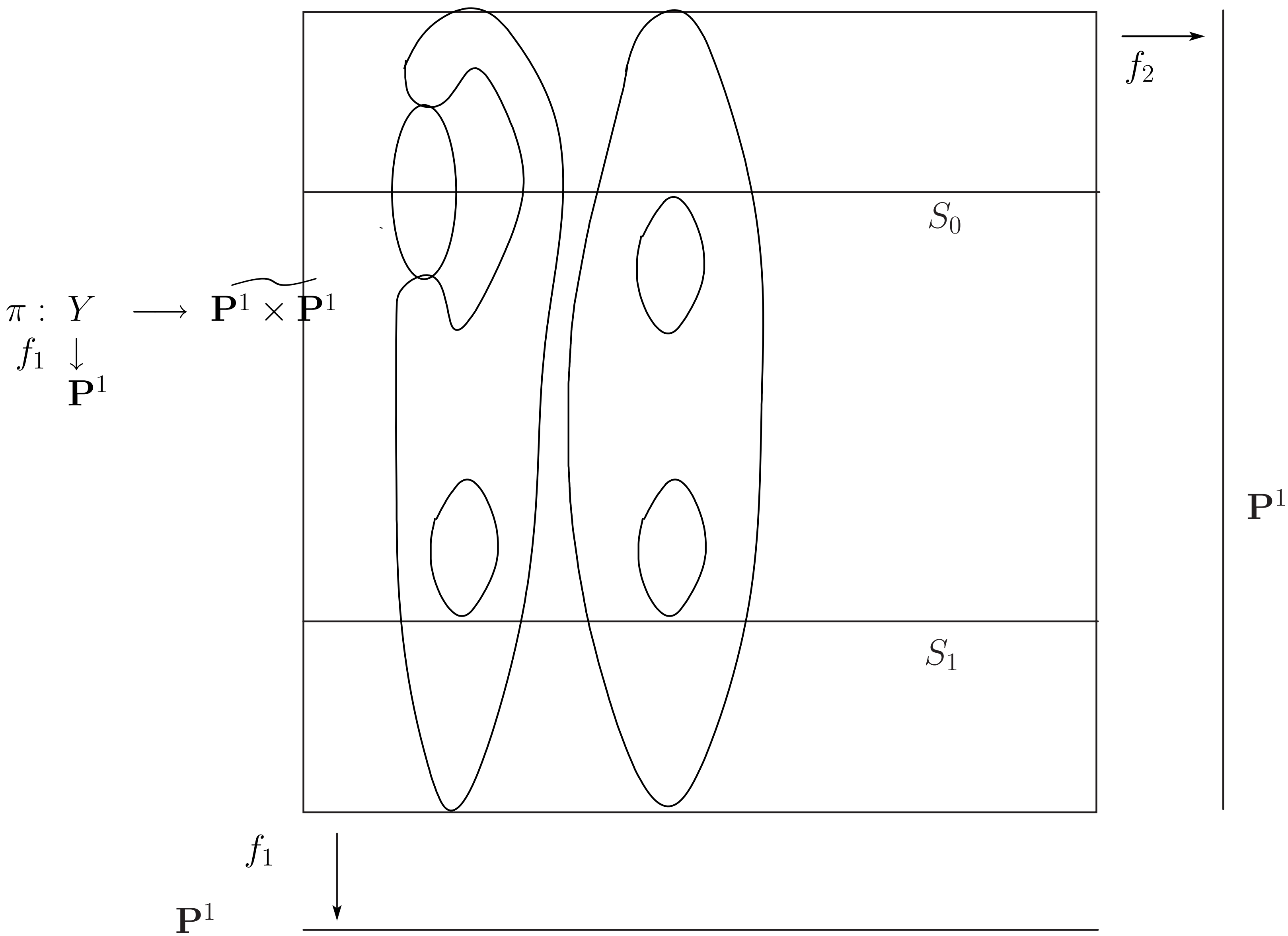
$\mathbf{P}^1 \times \mathbf{P}^1$
$\cup$
B
type (6,2)



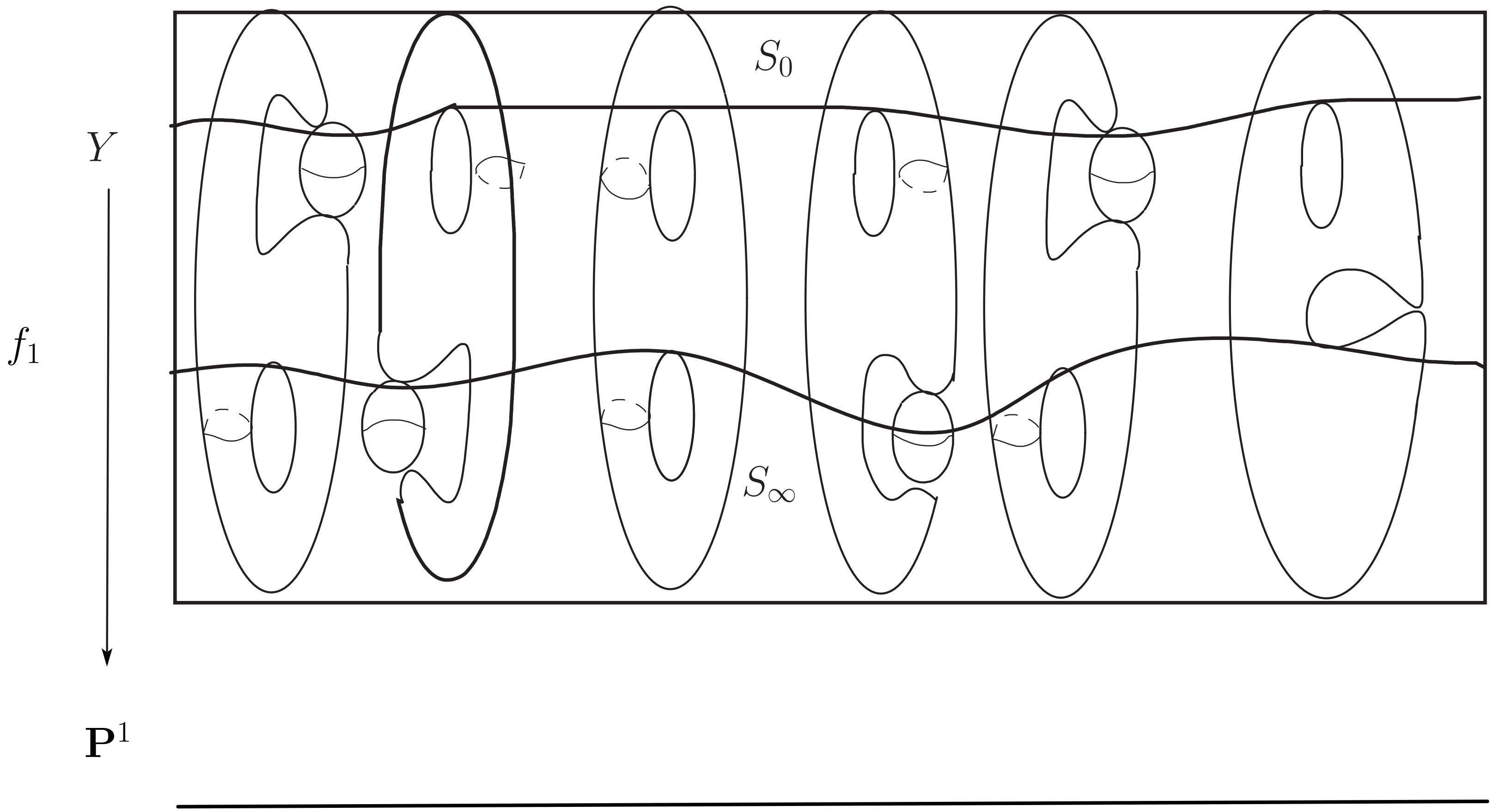
Our cases

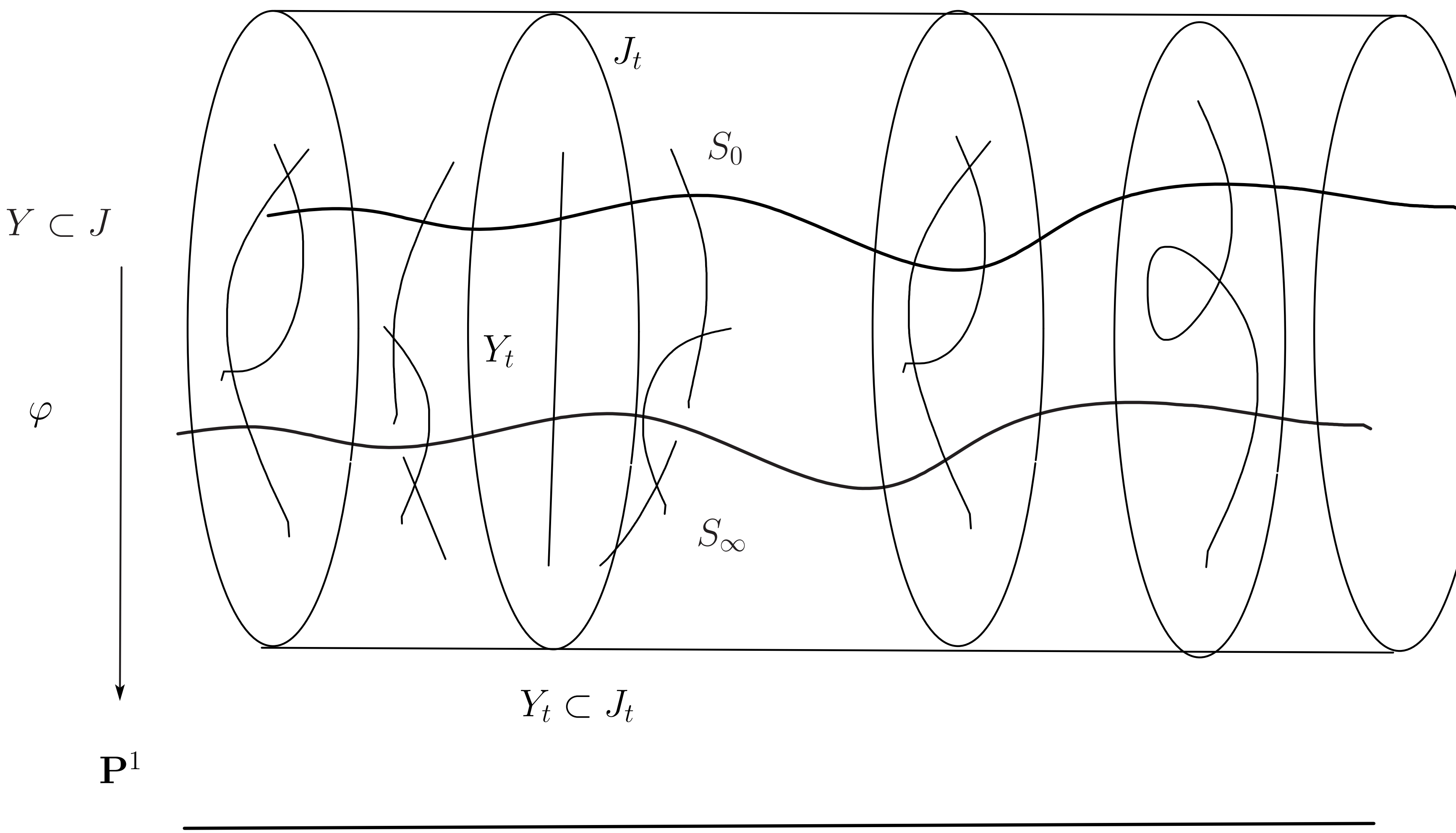
$$\mathbf{P}^1 \times \mathbf{P}^1$$
$$\cup$$
$$B = S_0 + S_1 + B_1$$





$\pi : Y \longrightarrow \widetilde{\mathbf{P}^1 \times \mathbf{P}^1}$
$f_1 \downarrow$
$\mathbf{P}^1$
f_2
S_0
S_1
$\mathbf{P}^1$
f_1
$\mathbf{P}^1$


S_0
Y
f_1
S_∞
$\mathbf{P}^1$



J_t
S_0
$Y \subset J$
Y_t
φ
S_∞
$Y_t \subset J_t$
$\mathbf{P}^1$

Degenerate fiber of J