Breaking GUTs in F-Theory

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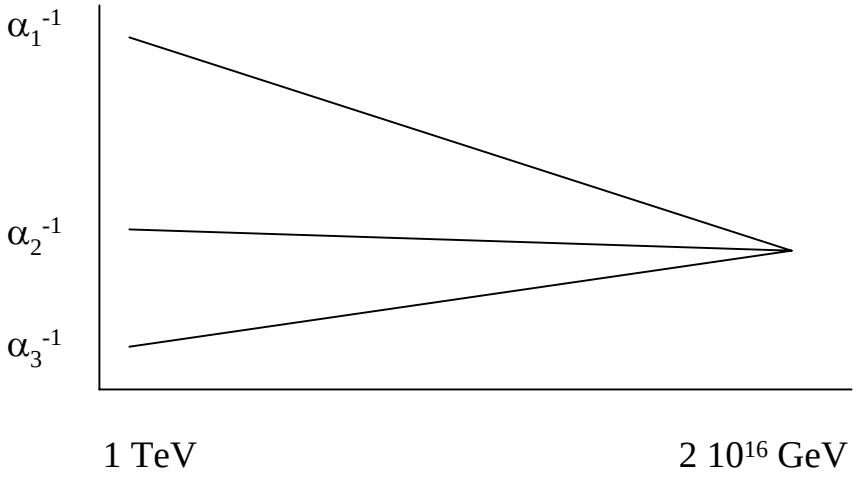
Based on: R. Donagi and MW, **arXiv:0802.2969** [hep-th]
MW & RD, in progress

Motivation: gauge coupling unification.



Unification at the GUT scale?

$$SU(3)\times SU(2)\times U(1) \quad \subset \quad SU(5) \quad \subset \quad SO(10) \quad \subset \quad E_6$$

But: traditional D-brane models have trouble with GUT groups.

• SO(10) GUT: no **16** (spinor representation)

• SU(5) GUT: top quark Yukawa from **10** x **10** x $\mathbf{5}_{h.}$

Since SU(5) $\rightarrow$ U(5), must be non-perturbative *and* order one.

Such restrictions may be evaded if we have exceptional gauge symmetry.

Exceptional gauge symmetry in string theory:

10 dimensions — Heterotic string

9 dimensions — Strongly coupled Type I'

8 dimensions — F-theory on ALE $\Longrightarrow$ CY_4 compactifications

7 dimensions — M-Theory on ALE $\Longrightarrow$ G_2 compactifications

Existence? Does *not* follow from Het/M duality

(Eg. D-terms are not isomorphic)

6 dimensions — IIa on ALE/IIb with NS5

No chiral matter in 4D

Stringy cosmic strings in four dimensions

Consider $$S^{(4)} = \int_{R^4} \sqrt{g} \left[R - \frac{1}{2} \frac{d^2\tau}{\mathrm{Im}(\tau)^2} \right]$$

This is a reduction of $$S^{(6)} = \int_{R^4 \times T^2} \sqrt{g}\ R$$

Here τ is the complex structure of T^2, defined up to PSl(2,Z) transformations.

Recipe for constructing solitons (vortices, "cosmic strings") of $S^{(4)}$:

Greene/Shapere/Vafa/Yau

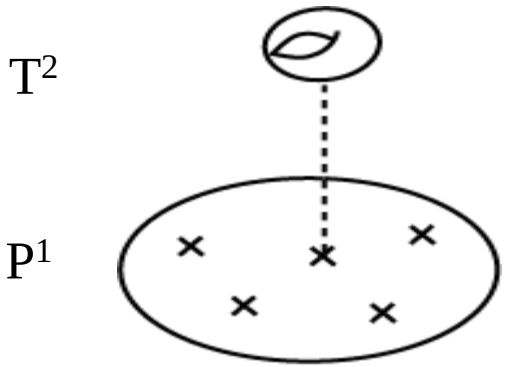
Choose complex coordinate z on R^4

$$ds^2 = -dt^2 + dx^2 + e^{\varphi(z,\bar{z})} dz d\bar{z}$$

$$e^{\varphi} = \mathrm{Im}(\tau) \eta^2 \bar{\eta}^2 \left| \prod_{i=1}^{N} (z - z_i)^{-1/12} \right|^2 \quad \Rightarrow \quad \text{CY metric on } (T^2 \rightarrow M_4)$$

$$\bar{\partial}\tau(z,\bar{z}) = 0, \qquad j(\tau(z)) = \frac{P(z)}{Q(z)} \quad \text{rational}$$

Main example: elliptically fibered K3 surface



24 singular fibers

Monodromy around singular fibers:

$$\tau \to e^{Q}\tau \qquad Q = \begin{pmatrix} 0 & p \\ -q & 0 \end{pmatrix}$$

$\Rightarrow$ 24 cosmic strings on R^2 x P^1

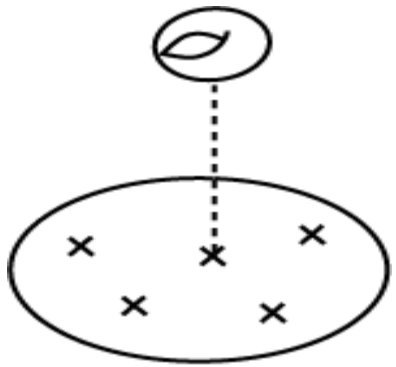
Seven-branes in IIb string theory

$$S^{(10)} = \int_{R^{10}} \sqrt{g} \left[R - \frac{1}{2} \frac{d^2\tau}{\mathrm{Im}(\tau)^2} \right] + \ldots$$

Here $\tau = \frac{i}{g_s} + a_{RR}$ and is defined mod PSl(2,Z)

$\Longrightarrow$ Use stringy cosmic string to construct BPS 7-branes solutions in IIb string theory

IIb $\left\{ \begin{array}{l} T^2 \\ P^1 \\ R^8 \end{array} \right.$



Monodromy around singular fibers:

$$\tau \to K_{[p,q]}\tau \qquad K_{[p,q]} = \begin{pmatrix} 1+pq & p^2 \\ -q^2 & 1-pq \end{pmatrix}$$

$\Longrightarrow$ (p,q) 7-brane

7-branes are *mutually non-local* if $\begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} \neq 0$

This 12-dimensional construction is called *F-Theory*

Abelian Gauge fields

The U(1) gauge fields on 7-branes are collective coordinates constructed from $B^{(2)}_{NS,}\ C^{(2)}_{RR}$

In turn, $B^{(2)}_{NS,}\ C^{(2)}_{RR}$ are most naturally encoded in a 3-form $C^{(3)}$ living in 12 dimensions, with one leg on the T^2:

$$C^{(3)} \sim (C_{RR} - \tau\ B_{NS}) \wedge (dx + \tau\ dy) + c.c.$$ Gukov/Vafa/Witten

Therefore U(1) gauge fields come from harmonic 2-forms with one leg on T^2:

$$C^{(3)}_{\mu\nu\lambda} = A^I_\mu \wedge \omega_{I,\nu\lambda}$$

The divergence $G_4 = dC_3$ is often called the *G-flux*.

Non-abelian Gauge fields

Colliding 7-branes lead to ALE singularities of the T^2-fibration

ADE singularity $\Longrightarrow$ ADE enhanced gauge symmetry

The non-abelian gauge bosons are BPS string junctions

How do we get (charged) chiral matter in IIb string theory?

Take two 7-branes intersecting over $R^4 \times \Sigma$ with fluxes F_1 and F_2 (Σ a Riemann surface)

Net number of chirals on R^4 charged under $U(N_1) \times U(N_2)$ is given by

$$\text{Net chiral} = \frac{1}{2\pi}\int_{\Sigma} F_1 - F_2$$

If N=1 SUSY in 4D is preserved, can also compute actual number of chirals (not just net number)

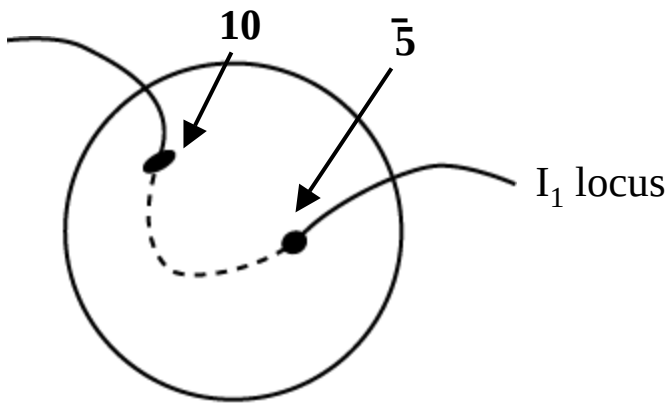
In order to generalize this to F-theory, need to address the following:

* Branes are not necessarily mutually local (get more interesting matter representations)

* Allowed 7-brane worldvolume fluxes are encoded in G-flux

For details, see Donagi/Wijnholt (see also Beasley/Heckman/Vafa, Watari et al.)

These ingredient allow one to construct GUT models with SU(5) (or SO(10)) gauge group



SU(5) brane wrapping compact S_4

Bulk chiral matter on SU(5) brane also allowed; see Donagi/Wijnholt, Beasley/Heckman/Vafa

But how does one break the GUT group?

Breaking the GUT group:

* Adjoint matter: in principle allowed in F-theory, but undesirable 4D phenomenology

* Discrete Wilson lines on 7-brane worldvolume. But discrete symmetries have fixed points, leading to singularities on the 7-brane worldvolume

* GUT breaking by U(1) fluxes

  Allowed a priori, though some subtleties in implementation (in progress).

  But at least toy models with 3 net generations, SM gauge group and primitive G-flux seem to be available.

Some of the issues with breaking by U(1) fluxes (in progress)

* coupling to closed string axions:

$$\mathcal{L} \approx \int_{\mathcal{R}^4} d^4x \, (\Pi_M A_\mu^Y + \partial a_M)^2$$

where $\Pi_M = \int_{Y_4} G \wedge \omega_Y \wedge \beta_M$ and $C_{RR}^{(4)} = a_M \wedge * \beta^M$

So $A_\mu{}^Y$ picks up a mass unless $\Pi_M = 0$ for all M

$\Longrightarrow$ Topological constraint on UV completion (i.e. compactification)

This is similar to Buican/Malyshev/Morrison/Verlinde/MW.

* Simple suggestion: $G \sim \alpha \wedge \omega_Y, \quad \alpha \in CoKer\,(i^*) : H^{1,1}(Y_4) \to H^{1,1}(S)$

Ruled out: gives wrong spectrum.

Need more general G-fluxes, similar to heterotic U(n) x U(1) constructions; it appears they can be chosen primitive (so that the D-terms are satisfied).

* Need higher derivative ($\mathrm{Tr}(F^4)$) and KK threshold corrections to be small.

Summary:

- All the ingredients for GUT model building are available in F-theory:

  GUT groups, chiral matter, Yukawa couplings, GUT breaking.

- Local model building $\Longrightarrow$ more flexible than heterotic string

- Toy models, but completely realistic model not yet constructed

- Coupling to 4D gravity while satisfying phenomenological constraints will be challenging.