A New Path to SO(10) Unification and Proton Decay

[PN]

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Based on work with Kaladi Babu, Ilia Gogoladze, and Raza Syed

Contents

LHC, Neutrino Masses, GUTS/strings

- LHC will directly probe phenomena up to scales about 10 times the Fermi scale

$$G_F^{-1/2} \sim 250 \text{ GeV}$$

- Neutrino masses via Type I seesaw allow us to probe physics at an intermediate scale M_{RR} where

$$m_\nu = \frac{m_D^2}{M_{RR}}, \quad M_{RR} \sim 10^{12-14}\text{GeV}$$

- GUTS/strings allow a probe of scales even larger

$$M \sim 10^{16-18} \text{ GeV}$$

Paths to unification

$$\textbf{a. Standard model} \leftrightarrow \textbf{GUTs}$$

$$\updownarrow$$

$$\textbf{b. Standard model} \leftrightarrow \textbf{strings}^1$$

a. GUTS

- SU(5) models
  - Minimal model ruled out
  - Non-minimal still viable: Planck slop, additional Higgses
  - Flipped SU(5) models
- E(6) models:
  - Typically too many exotics.
- SO(10) models
  - These are phenomenologically the most successful of the grand unified models.

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[1] Recent progress: Bouchard, Donagi, ..Cvetic, Shiu, Bloomenhagen, Kors, Lust, ..Lebedev, Nilles, Raby,..; F-theory compactifications: Vafa talk.

The Conventional $SO(10)$ schemes

- In conventional $SO(10)$ models several Higgs reps needed. The arbitrariness of the Higgs sector allows for a huge number of possibilities for building models. So in this regard $SO(10)$ does poorly relative to $SU(5)$.
- The $SO(10)$ models can be roughly classified into the classes
  - Models with small Higgs representation:
    $\mathbf{45}$-plet to break $SO(10)$ in $B-L$ direction
    $\mathbf{16}+\overline{\mathbf{16}}$ to break $B-L$
    $\mathbf{10}$-plets to break EW symmetry.
  - Models with large Higgs representations:
    $\mathbf{120}, \mathbf{126}+\overline{\mathbf{126}}, \mathbf{210}, \mathbf{10}, \mathbf{54}$
- An ideal scenario
  - Family unification
  - Unification in the Higgs sector

Family Unification

Family unification is not possible in $\boldsymbol{SO(10)}$. Some suggested models for family unification include

- $\boldsymbol{SO(18)}$: In $\boldsymbol{SO(10)}$ decomposition, the $\mathbf{256}$ dimensional (semi)spinor representation of $\boldsymbol{SO(18)}$ decomposes into eight $\mathbf{16}$ plet representations and eight $\overline{\mathbf{16}}$ representations. One needs to split them, making some superheavy and others light.
  Senjanovic, Zee, Wilczek, ..
- E(8): In $\boldsymbol{SU(3) \times E_6}$ decomposition

$$\mathbf{248 = (8,1) + (1,78) + (3,27) + (\bar{3},\overline{27})}$$

  One can unify three families within one representation, but three mirror families lead to difficulties.
  Bars, Gunaydin; Ong; Buchmuller, Napoly; Adler, ..
- String models do better as far as family unification is concerned as one puts in a constraint $(\boldsymbol{n_f - n_{mf} = 3})$, although usually one still relies on the survival hypothesis.

Unification in the Higgs sector

It is possible to unify the Higgs sector in a single irreducible representation $\mathbf{144}$ for non-susy $SO(10)$. For SUSY $SO(10)$ one can unify the Higgs sector with $\mathbf{144}(\Psi^{\mu}_{+}) + \overline{\mathbf{144}}(\Psi^{\mu}_{-})$.

- Under $SU(5) \times U(1)$:

$$\mathbf{144} = \bar{\mathbf{5}}_{\mathbf{3}} + \mathbf{5}_{\mathbf{7}} + \mathbf{10}_{-\mathbf{1}} + \mathbf{15}_{-\mathbf{1}} + \underline{\mathbf{24}_{-\mathbf{5}}} + \mathbf{40}_{-\mathbf{1}} + \overline{\mathbf{45}}_{-\mathbf{3}}$$

- Typically when the $\mathbf{24}$ plet of $SU(5)$ develops a VEV, $SU(5)$ will break to the SM gauge group.
- However, there is an extra $U(1)$ factor and we need to reduce the rank. Since the the 24-plet has a $U(1)$ charge, once the 24-plet gets a VEV, there is a change in the rank.
- One needs an explicit construction to exhibit this breaking.

Spontaneous breaking of SO(10)

The superpotential with $\mathbf{144}$ and $\overline{\mathbf{144}}$-plets

$$W = M(144 \times \overline{144}) + \frac{\lambda_{45_1}}{M'}(144 \times \overline{144})_{45_1}(144 \times \overline{144})_{45_1}$$
$$\frac{\lambda_{45_2}}{M'}(144 \times \overline{144})_{45_2}(144 \times \overline{144})_{45_2}$$
$$+\frac{\lambda_{210}}{M'}(144 \times \overline{144})_{210}(144 \times \overline{144})_{210}$$

$$< 24_{144} >= q\ diag(2,2,2,-3,-3).$$
$$< 24_{\overline{144}} >= p\ diag(2,2,2,-3,-3).$$

Symm breaking constraint

$$\frac{MM'}{qp} = 116\lambda_{45_1} + 7\lambda_{45_2} + 4\lambda_{210}.$$

D flatness requires $p = q$.
With the above $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$.

$$SU(2)\times U(1)_Y \rightarrow U(1)_{em}$$

$\mathbf{144}(\overline{\mathbf{144}})$ contain fields which have the same quantum numbers as the SM Higgs doublets. In $SU(5)$ decomposition

$$144 \supset \; Q_i(\bar{5}) + Q^i(5) + Q^k_{ij}(\overline{45})$$

$$\overline{144} \supset P_i(\bar{5}) + P^i(5) + P^{ij}_k(45)$$

- The 45 plet contains $SU(2)$ doublets as can be seen in its $SU(3)\times SU(2)\times U(1)_Y$ decomposition

$$45 = (1,2)(3) + (3,1)(-2) + (3,3)(-2) + (\bar{3},1)(8)$$
$$+(\bar{3},2)(-7) + (\bar{6},1)(-2) + (8,2)(3)$$

- 3 pairs of Higgs doublets. One can arrange one pair of Higgs doublets to be light while the Higgs triplets remain heavy.
- mass^2 of the Higgs that couples with the top turns negative by RG effects breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$.

Quark, lepton, neutrino masses

Matter-Higgs couplings are at least quartic

$$(\mathbf{16}\times\mathbf{16})_{\mathbf{10}}(\mathbf{144}\times\mathbf{144})_{\mathbf{10}},\quad (\mathbf{16}\times\mathbf{16})_{\mathbf{10}}(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{10}}$$

$$(\mathbf{16}\times\mathbf{16})_{\mathbf{120}}(\mathbf{144}\times\mathbf{144})_{\mathbf{120}},\quad (\mathbf{16}\times\mathbf{16})_{\mathbf{120}}(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{120}}$$

$$(\mathbf{16}\times\mathbf{16})_{\overline{\mathbf{126}}}(\mathbf{144}\times\mathbf{144})_{\mathbf{126}},\quad (\mathbf{16}\times\mathbf{16})_{\overline{\mathbf{126}}}(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{126}},\cdots$$

Quark-lepton-neutrino masses arise as follows

$$\mathbf{up-quark:}\qquad (\mathbf{10}_M\mathbf{10}_M)\,\frac{\mathbf{24}_H}{M'}\mathbf{5}_H$$

$$\mathbf{down-quark-lepton:}\qquad (\mathbf{10}_M\bar{\mathbf{5}}_M)\,\frac{\mathbf{24}_H}{M'}\bar{\mathbf{5}}_H$$

$$\mathbf{RR}-\boldsymbol{\nu}:\qquad (\mathbf{1}_M\mathbf{1}_M)\,\frac{\mathbf{24}_H}{M'}\mathbf{24}_H$$

$$\mathbf{LR}-\boldsymbol{\nu}:\qquad (\bar{\mathbf{5}}_M\mathbf{1}_M)\,\frac{\mathbf{24}_H}{M'}(\mathbf{5}_H,\mathbf{45}_H)$$

$$\mathbf{LL}-\boldsymbol{\nu}:\qquad (\bar{\mathbf{5}}_M\bar{\mathbf{5}}_M)\,\frac{\mathbf{5}_H}{M'}\mathbf{5}_H$$

Size estimates for quark, charged lepton, neutrino masses

up-quark	$(\mathbf{10}_M \mathbf{10}_M)\,\frac{\mathbf{24}_H}{M'}\mathbf{5}_H$	Yukawa. $\mathbf{10^{\pm 1}}$ GeV
down-quark-lepton	$(\mathbf{10}_M \bar{\mathbf{5}}_M)\,\frac{\mathbf{24}_H}{M'}\bar{\mathbf{5}}_H$	Yukawa. $\mathbf{10^{\pm 1}}$ GeV
RR-Majorana $\boldsymbol{\nu}$ Type I seesaw	$(\mathbf{1}_M \mathbf{1}_M)\,\frac{\mathbf{24}_H}{M'}\mathbf{24}_H$	$\mathbf{10^{13\pm 1}}$GeV
LR- Dirac $\boldsymbol{\nu}$ Type I seesaw	$(\bar{\mathbf{5}}_M \mathbf{1}_M)\,\frac{\mathbf{24}_H}{M'}(\mathbf{5}_H, \mathbf{45}_H)$	Yukawa. $\mathbf{10^{\pm 1}}$ GeV
LL-$\boldsymbol{\nu}$ Type II seesaw	$(\bar{\mathbf{5}}_M \bar{\mathbf{5}}_M)\,\frac{\mathbf{5}_H}{M'}\mathbf{5}_H$	$(\mathbf{1 - 10^{-3}})$ eV

Estimate of sizes with

$$q = p \simeq 10^{16}\mathrm{GeV}, \quad M' \simeq 10^{18}\ \mathrm{GeV}$$

$$< \mathbf{5}_H > \simeq < \bar{\mathbf{5}}_H > \simeq 10^2\mathrm{GeV}$$

Fermion generation mass hierarchy

The charged fermions lie in a mini landscape: $\mathbf{10^{-3} - 10^{2}}$ GeV.

- One possibility is that the masses of the first two generations vanish at the cubic level and arise from higher dimensional operators which are suppressed by the Planck mass. This is the picture that emerges with the $\mathbf{144}-$ plet couplings. Here the Yukawas will be suppressed typically by

$$\epsilon = (q/M') \simeq 10^{-2}$$

  and thus be naturally small.
- What about the third generation Yukawas? Here one would like cubic interactions. With the 16-plet of matter and $\mathbf{144}$ plet of Higgs, no cubic interactions are allowed. However, one can get cubic interactions allowing for new matter in $\mathbf{10}_M$ and $\mathbf{45}_M$: Babu, Gogoladze, PN, Syed.

$$M^{(10)}\mathbf{10}_M^2 + M^{(45)}\mathbf{45}_M^2 + f^{(10)}\mathbf{16}_M.\mathbf{10}_M.\mathbf{144}_H + f^{(45)}\mathbf{16}_M.\mathbf{45}_M.\overline{\mathbf{144}_H}$$

The bottom quark and τ lepton masses

The physical $b_{L,R}$ fields are linear combination of fields as follows

$$b_L : \quad {}^{(10_{16})}\mathsf{b}_L^{\alpha}, \quad {}^{(10_{45})}\mathsf{b}_L^{\alpha}, \quad {}^{(5_{10})}\mathsf{b}_L^{\alpha}$$

$$\bar{b}_R : \quad {}^{(\bar{5}_{16})}\overline{\mathsf{b}}_{R\alpha}, \quad {}^{(\bar{5}_{10})}\overline{\mathsf{b}}_{R\alpha}, \quad {}^{(\overline{10}_{45})}\overline{\mathsf{b}}_{R\alpha}$$

They mix via the matrix

$$\begin{pmatrix} 0 & m_b'' & M_B^{(10)} \\ m_b' & 0 & M^{(10)} \\ M_B^{(10)} & M^{(10)} & 0 \end{pmatrix}$$

where $m_b' \sim f^{(10)} < H_d >$, and $m_b \sim f^{(45)} < H_d' >$, and $(M_B^{(10)}, M^{(10)})$ are GUT scale size. Diagonalization gives one light eigenvalue of size $O(m_b', m_b'')$ and two eigenvalues of size $O(M_B^{(10)}, M^{(10)})$ which are super heavy and eliminated from the spectrum. A similar analysis holds for τ.

The top quark mass

The physical $t_{L,R}$ fields are linear combination of fields as follows

$$t_L : \quad {}^{(10_{16})}\mathrm{t}_L^{\alpha}, \quad {}^{(10_{45})}\mathrm{b}_L^{\alpha}, \quad {}^{(\overline{10}_{45})}\mathrm{t}_L^{\alpha}$$

$$\bar{t}_R : \quad {}^{(10_{16})}\bar{\mathrm{t}}_{R\alpha}, \quad {}^{(10_{45})}\bar{\mathrm{t}}_{R\alpha}, \quad {}^{(\overline{10}_{45})}\bar{\mathrm{t}}_{R\alpha}$$

They mix via the matrix

$$\begin{pmatrix} 0 & m_t' & M_U^{(10)} \\ m_t' & 0 & M^{(45)} \\ M_U^{(45)} & M^{(45)} & 0 \end{pmatrix}$$

where $m_t' \sim f^{(45)} < H_d >$, and $(M_U^{(45)}, M^{(45)})$ are GUT scale size. Diagonalization gives one light eigenvalue of size $O(m_t')$ and two eigenvalues of size $O(M_U^{(45)}, M^{(45)})$ which are super heavy and eliminated from the spectrum.

$b-\tau$ and $b-t-\tau$ unification

The $b-t-\tau$ unification is modified from the standard $SO(10)$ picture

- In the standard $SO(10)$ models where the electroweak symmetry breaking occurs via the 10-plet one has the relation at the GUT scale

$$h_\tau = h_b = h_t$$

  which implies large $\tan\beta =< H_2/H_1 >$ to explain the large ratio of the top to the bottom mass.

- For the 144 plet breaking and two additional Yukawas, there is no firm prediction on $\tan\beta$. However, low $\tan\beta$ does allow $b-t-\tau$ unification. One relation that emerges in certain limits is

$$|h_\tau| \approx h_b \approx \frac{z}{6}|h_t|$$

  where $z = O(1)$. $b-t-\tau$ unification can be achieved with a $\tan\beta$ as low as 10 instead of $\tan\beta = 50$.

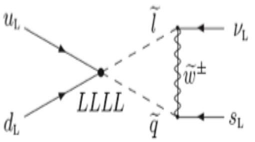
Proton decay in GUTs and Strings

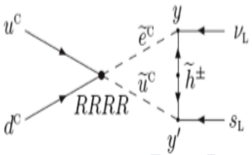
Proton decay is an important constraint on unified models. For example

- GG model was eliminated by the p-decay constraint. This constraint was from the vector lepto-quark exchange.
- The minimal SUSY $SU(5)$: here lepto-quark exchange is fine, but the model is eliminated by $B\&L$ violating Higgsino mediated proton decay.
- B& L violating squark exchange (arising from operators QLD^c, $U^cD^cD^c$, LLE^c) kills many GUT and string models. It can be eliminated by imposing R parity conservation which, however, still allows B& L violating Higgsino mediated proton decay.

  In fact in SUSY models and many string models the Higgsino mediated proton decay turns out to be the most serious constraint.

$$LLLL : C_{ikl}(Q_i.Q_i)(Q_k.L_l)/M_T$$
$$RRRR : C'_{ijkl}u_i^C e_j^C u_k^C d_l^C /M_T$$





Higgsino mediated proton decay

GUT/string models contain color triplet (anti-triplet) Higgsinos $H_T^a(H'_{Ta})$ $[Q = -1/3(1/3)]$ which generate both $LLLL$ and $RRRR$ B&L violating interactions. These have the couplings

$$H'_T \mathcal{M} H_T + H'_T K + H_T J$$

$$J_{q\alpha} = f^{(1)}_{q\acute{a}\acute{b}} \epsilon_{\alpha\beta\gamma} Q^{\beta}_{\acute{a}} Q^{\gamma}_{\acute{b}} + f^{(2)}_{q\acute{a}\acute{b}} U^C_{\acute{a}\alpha} E^C_{\acute{b}}$$

$$K^{\alpha}_p = f^{(1)'}_{p\acute{a}\acute{b}} Q^{\alpha}_{\acute{a}} L_{\acute{b}} + f^{(2)'}_{p\acute{a}\acute{b}} \epsilon^{\alpha\beta\gamma} U^C_{\acute{a}\beta} D^C_{\acute{b}\gamma}.$$

Additionally often one has Higgsino triplets (anti-triplets) $\tilde{H}_T^a(\tilde{H}_{Ta})$ $[Q = -4/3(4/3)]$ with couplings

$$\tilde{H}'_T \tilde{\mathcal{M}} \tilde{H}_T + \tilde{H}'_T \tilde{K} + \tilde{H}_T \tilde{J}$$

$$\tilde{J}_{q'\alpha} = \tilde{f}_{q'\acute{a}\acute{b}} D^C_{\acute{a}\alpha} E^C_{\acute{b}}, \;\; \tilde{K}^{\alpha}_{p'} = \tilde{f}'_{p'\acute{a}\acute{b}} \epsilon^{\alpha\beta\gamma} U^C_{\acute{a}\beta} U^C_{\acute{b}\gamma}.$$

which generate only the $RRRR$ type B&L violating interactions.

Conditions for suppression Higgsino mediated proton decay

Higgsino mediated proton decay including that from Planck scale corrections can be suppressed by the conditions

(PN, Raza Syed, PRD 77, 015015 (2008))

$$\boldsymbol{LLLL}: \; (U\mathcal{M}V^T)^{-1}_{11} + \Lambda_{\rm Planck} = 0$$

$$\boldsymbol{RRRR}: \; (U\mathcal{M}V^T)^{-1}_{11} + (\tilde{U}\tilde{\mathcal{M}}\tilde{V}^T)^{-1}_{11} + \tilde{\Lambda}_{\rm Planck} = 0$$

$U, V(\tilde{U}, \tilde{V})$ etc take us to the basis where only $H_1, H_1'(\tilde{H}_1 \tilde{H}_1')$ couple with matter. There are various ways to satisfy the constrains.

- Couplings to one or both of the Higgsino triplets vanish by a symmetry and no dim 5 operators are generated.
- All sfermions heavy such as in split SUSY.
- The cancellation mechanism: it can operate if there are more than one source of Higgsino mediation.

Examples of cancellations for suppression of Higgsino mediated proton decay

- An $SU(5)$ model with the Higgs structure

$$\mathbf{5}_H, \bar{\mathbf{5}}_H, \mathbf{24}_H, \mathbf{45}_H, \overline{\mathbf{45}}_H$$

  Cancellations between $(\mathbf{5}_H, \bar{\mathbf{5}}_H)$ and $(\mathbf{45}_H, \overline{\mathbf{45}}_H)$ contributions suppress Higgsino mediated proton decay.
- The above cancellation possibility arises naturally in $\mathbf{144}(\overline{\mathbf{144}})$ plet couplings

$$\mathbf{144} \supset \mathbf{5(3)} + \bar{\mathbf{5}}\mathbf{(7)} + \overline{\mathbf{45}}\mathbf{(3)}$$

$$\overline{\mathbf{144}} \supset \bar{\mathbf{5}}\mathbf{(-3)} + \mathbf{5(-7)} + \mathbf{45(-3)}$$

  - An internal cancellation can occur among the contributions from two pairs of color triplets (anti-triplets) from $\mathbf{5}(\bar{\mathbf{5}})$ and a pair of color triplets (anti-triplets) from $\mathbf{45}(\overline{\mathbf{45}})$.
  - Proton decay can be suppressed to current limit $p \to \bar{\nu} K^+ > 2.3 \times 10^{33}$yr, and may be accessible in proposed experiments such as Hyper K, MEMPHYS, or at DUSEL.

Field theoretic description for $SO(10)$ analysis

A convenient field theoretic description of $SO(10)$ computations is in terms of $SU(5)$ oscillators [more generally to decompose SO(2N) couplings in representations of $SU(N)$ oscillators]. Consider harmonic oscillators

$$\{b_i, b_j^\dagger\} = \delta_{ij}, \quad \{b_i, b_j\} = 0; i, j = 1 - 5$$

Define SO(10) operators $\Gamma_\mu (\mu = 1, 2, .., 10)$

$$\Gamma_{2i} = (b_i + b_i^\dagger), \quad \Gamma_{2i-1} = -i(b_i - b_i^\dagger)$$

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}$$

In terms of the oscillators one

$$\overline{\mathbf{16}} : |\Psi_{(-)} >= (\mathrm{P} + b_i^\dagger b_j^\dagger \mathrm{P}^{ij} + \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger \mathrm{P_i})|0 >$$

(Mohapatra, Sakita: Wilczek, Zee)

A basic theorem for analysis of $SO(2n)$ vertices

PN, Raza Syed: PLB 506, 68(2001); Nucl. Phys. B618, 138(2001); Nucl. Phys. B676, 64(2004)

Carry out a Wick type vertex expansion in $SU(n)$ oscillators using creation and desctruction operators

$$\Gamma_\mu \Gamma_\nu \Gamma_\lambda .. \Gamma_\sigma \phi_{\mu\nu\lambda..\sigma} = b_i^\dagger b_j^\dagger b_k^\dagger .. b_n^\dagger \phi_{c_i c_j c_k ... c_n}$$

$$+(b_i b_j^\dagger b_k^\dagger .. b_n^\dagger \phi_{\bar{c}_i c_j c_k ... c_n} + \ perms)$$

$$+(b_i b_j b_k^\dagger .. b_n^\dagger \phi_{\bar{c}_i \bar{c}_j c_k .. c_n} + \ perms)$$

$$+.... + (b_i b_j b_k .. b_{n-1}^\dagger b_n \phi_{c_i c_j c_k .. c_{n-1} c_n} + \ perms) +$$

$$+ b_i b_j b_k ..... b_n \phi_{\bar{c}_i \bar{c}_j \bar{c}_k ... \bar{c}_n}$$

where one must take into account all possible permutations of b and $b^\dagger$ in writing the expansion. The $\phi_{c_i c_j c_k ... c_n}$ etc then are reducible tensors of $SU(N)$ which can be further decomposed into irreducible ones.

$\mathbf{16} - \mathbf{16} - \mathbf{1\overline{2}6}$ coupling

$$\frac{1}{5!} f_{ab} \tilde{\psi}_a B \Gamma_\mu \Gamma_\nu \Gamma_\lambda \Gamma_\rho \Gamma_\sigma \psi_b \Delta_{\mu\nu\lambda\rho\sigma}$$

$\Delta_{\mu\nu\lambda\rho\sigma}$ is 252 dimensional and decomposable into $\overline{\mathbf{126}} + \mathbf{126}$

$$\begin{pmatrix} \bar{\phi}_{\mu\nu\lambda\rho\sigma} \\ \phi_{\mu\nu\lambda\rho\sigma} \end{pmatrix} = \frac{1}{2}(\delta_{\mu\alpha}\delta_{\nu\beta}\delta_{\rho\gamma}\delta_{\lambda\delta}\delta_{\sigma\theta} \pm \frac{i}{5!}\epsilon_{\mu\nu\rho\lambda\sigma\alpha\beta\gamma\delta\theta})\Delta_{\alpha\beta\gamma\delta\theta}$$

$\overline{\mathbf{126}} = \mathbf{1}(h) + \mathbf{5}(h^i) + \mathbf{\bar{1}0}(h_{ij}) + \mathbf{15}(h_S^{ij}) + \mathbf{\bar{4}5}(h_{ij}^k) + \mathbf{50}(h_{rs}^{ijk})$. A direct analysis gives

$$W^{(1\bar{2}6)} = i f_{ab}^{(+)} \frac{\sqrt{2}}{\sqrt{15}} [-\sqrt{2} M_{0a} M_{0b} h - \sqrt{3} M_{0a} M'_{ib} h^i$$
$$+ M_{0a} M_b^{ij} h_{ij} - \frac{1}{8\sqrt{3}} M_a^{ij} M_b^{kl} h^m \epsilon_{ijklm}$$
$$- h_S^{ij} M'_{ia} M'_{jb} + M_a^{ij} M'_{bk} h_{ij}^k - \frac{1}{12\sqrt{2}} \epsilon_{ijklm} M_a^{lm} M_b^{rs} h_{rs}^{ijk}]$$

Field theoretic description of 144 -plet

To generate a 144 ($\overline{\mathbf{144}}$) we start with a unconstrained vector -spinor $\mathbf{160}(\overline{\mathbf{160}})$ and then obtain a constrained vector-spinor.

$$\mathbf{160} : |\Psi'_{(+)\mu}> = (\mathbf{P}_\mu + b_i^\dagger b_j^\dagger \mathbf{P}_\mu^{ij} + \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger \mathbf{P}_{i\mu})|0>$$

$\mathbf{144}, \overline{\mathbf{144}}$ ($|\Psi_{(\pm)\mu}>$) are constrained vector spinors

$$\Gamma_\mu |\Psi_{(\pm)\mu}> = |\Psi_\mp> = 0$$

The constraint requires that $\mathbf{16}_\mp$ fields must be removed from the $\mathbf{160}_\pm$ reducing it to $\mathbf{144}_\pm$.

Babu, Gogoladze, PN, Syed; PN, Syed

Vector-spinor continued

To get the $\overline{\mathbf{144}}$ spinor impose $\mathbf{\Gamma}_\mu|\mathbf{\Psi}_{(-)\mu}>= \mathbf{0}$ which gives

$$\overline{\mathbf{144}} = \overline{\mathbf{144}}^n(\mathbf{79}) + \overline{\mathbf{144}}_n(\mathbf{65})$$

In oscillator decomposition 144=79 + 65 components are

$$|\overline{\mathbf{144}}^n >= |0> \mathbf{P}^n + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \left[\epsilon^{ijklm}\mathbf{P}^n_{klm} - \frac{1}{6}\epsilon^{ijnlm}\mathbf{P}_{lm}\right]$$

$$+\frac{1}{24}\epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathbf{P}^n_i; \quad \mathbf{5} + \overline{\mathbf{10}} + \overline{\mathbf{40}} + \mathbf{24} = \mathbf{79}$$

$$|\overline{\mathbf{144}}_n >= |0> \mathbf{P}_n + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \left[\mathbf{P}^{ij}_n + \frac{1}{4}\left(\delta^i_n \mathbf{P}^j - \delta^j_n \mathbf{P}^i\right)\right]$$

$$+\frac{1}{24}\epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \left[\frac{1}{2}\mathbf{P}_{in} + \frac{1}{2}\mathbf{P}^{(S)}_{in}\right]; \quad \bar{\mathbf{5}} + \overline{\mathbf{15}} + \mathbf{45} = \mathbf{65}$$

PN, Raza Syed: JHEP, 02,(2006),022.
Gauge Couplings of vector-spinors

$$(\mathbf{144}^\dagger \times \mathbf{144} \times \mathbf{1}), \quad (\mathbf{144}^\dagger \times \mathbf{144} \times \mathbf{45}), (\overline{\mathbf{144}}^\dagger \times \overline{\mathbf{144}} \times \mathbf{1}), \quad (\overline{\mathbf{144}}^\dagger \times \overline{\mathbf{144}} \times \mathbf{45})$$

Higgs Sector Couplings

$$\begin{aligned}
&(\mathbf{144} \times \mathbf{144} \times \mathbf{1}), \quad (\mathbf{144} \times \mathbf{144} \times \mathbf{45}) \\
&(\mathbf{144} \times \mathbf{144} \times \mathbf{210}), \quad (\mathbf{144} \times \mathbf{144} \times \mathbf{10}), \quad (\mathbf{144} \times \mathbf{144} \times \mathbf{120}), \quad (\mathbf{144} \times \mathbf{144} \times \mathbf{126}) \\
&(\mathbf{144} \times \overline{\mathbf{144}})_{\mathbf{1}}(\mathbf{144} \times \overline{\mathbf{144}})_{\mathbf{1}}, \quad (\mathbf{144} \times \overline{\mathbf{144}})_{\mathbf{45}}(\mathbf{144} \times \overline{\mathbf{144}})_{\mathbf{45}} \\
&(\mathbf{144} \times \overline{\mathbf{144}})_{\mathbf{210}}(\mathbf{144} \times \overline{\mathbf{144}})_{\mathbf{210}}, \quad (\mathbf{144} \times \mathbf{144})_{\mathbf{10}}(\mathbf{144} \times \mathbf{144})_{\mathbf{10}} \\
&(\mathbf{144} \times \mathbf{144})_{\mathbf{120}}(\mathbf{144} \times \mathbf{144})_{\mathbf{120}}, \quad (\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{10}}(\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{10}} \\
&(\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{120}}(\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{120}}, \quad (\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{10}}(\mathbf{144} \times \mathbf{144})_{\mathbf{10}} \\
&(\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{120}}(\mathbf{144} \times \mathbf{144})_{\mathbf{120}}, \quad (\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\overline{\mathbf{126}}}(\mathbf{144} \times \mathbf{144})_{\mathbf{126}} \\
&(\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\overline{\mathbf{126}}}(\mathbf{16} \times \mathbf{16})_{\mathbf{126}}
\end{aligned}$$

Matter -Higgs couplings

$$\begin{aligned}
&(\mathbf{16} \times \mathbf{16})_{\mathbf{10}}(\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{10}}, (\mathbf{16} \times \mathbf{16})_{\mathbf{10}}(\mathbf{144} \times \mathbf{144})_{\mathbf{10}}, (\mathbf{16} \times \mathbf{16})_{\mathbf{120}}(\overline{\mathbf{144}} \times \overline{\mathbf{144}})_{\mathbf{120}} \\
&(\mathbf{16} \times \mathbf{16})_{\mathbf{120}}(\mathbf{144} \times \mathbf{144})_{\mathbf{120}}, (\mathbf{16} \times \mathbf{16})_{\overline{\mathbf{126}}}(\mathbf{144} \times \mathbf{144})_{\mathbf{126}}
\end{aligned}$$

Conclusions/prospects

- The use of a single representation to break SO(10) completely down to the SM gauge group symmetry and further down to the $SU(3)_C \times U(1)_{em}$ is rather attractive. It is more unified than previous works which use several Higgs representations.
- $\mathbf{144}$ is the simplest $SO(10)$ representation which does that. The next simplest is $\mathbf{560}$. In $SU(5) \times U(1)$ decomposition

  $$\mathbf{560} \supset \mathbf{24(-5)}.$$

  So it too can break $SO(10)$ down to the SM gauge group.
- The cancellation mechanism could operate in string models as well. While proton is stable in some string constructions dim 5 operators do surface in other models. The cancellation mechanism could be helpful in resolving the dim 5 proton decay problems in such models.