Dark Energy Theorems and Curious Corollaries

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& PJS, to appear

complement but NOT directly related to:

no-go theorems based on SuSY, SuGRA
or based on ε or η problems
or constructions leading to string landscape

more closely related to:

constraints on static deS and assuming SEC ($\rho+3p>0$):

G. Gibbons (1985)
J. Maldacena & C. Nunez (2001)
S. Giddings, S. Kachru and J. Polchinski (2002)
S. Giddings and A. Maharana (2005)
Carroll, Geddes, Hoffman, Wald (2002)

directly related to:

D. Wesley, arXiv:0802.2106
(establishes the methodology)

Theorem I:

Suppose fundamental physics described by::

1) compactifying from higher dimensions with 13>D>5

2) where higher D theory described by Einstein GR

3) where extra D is conformally Ricci-flat* and

$$ds^2 = e^{-2\Omega}(-dt^2 + a^2(t)\,dx^2) + e^{2\Omega} g_{ab} dy^a dy^b$$

4) and extra D is closed or orbi/orientifolded

5) null energy condition (NEC) is satisfied

. . . then $w = p/\rho$ must be > -1

Strategy in proving dark energy theorems

Do all calculations in Einstein frame so interpretation is unambiguous

Treat $T_{\mu\nu}$ space-space components as block diagonal

Then try to determine how higher D $\langle \rho + p_k \rangle_A$ & $\langle \rho + p_3 \rangle_A$

relate to the observed ρ^{4d} *and* p^{4d}

(which obey usual Friedmann eqs.)

Strategy in proving dark energy theorems

A-averaging

NEC $\quad T_{MN}\, n^M n^N \geq 0 \;\; for\;\; every\;\; null\;\; n^M$

A-averaged NEC $\quad \left\langle T_{MN}\, n^M n^N \right\rangle_A \equiv$

$$\left(\int T_{MN}\, n^M n^N \boxed{e^{A\Omega}}\, e^{2\Omega}\sqrt{g}\, d^k y\right) \Big/ \left(\int \boxed{e^{A\Omega}}\, e^{2\Omega}\sqrt{g}\, d^k y\right) \geq 0$$

N.B. If A-averaged NEC violated then NEC also violated (but not the converse)

Let's get to it . . .

$$consider \;\; general \;\; g_{ab}(t,y) \;\; and \;\; \Omega(t,y)$$

$$\frac{d}{dt} g_{ab} = \frac{2}{k}\xi\, g_{ab} + \sigma_{ab}$$

$$take \;\; linear \;\; combinations \; of \;\; G_{00} \;\; and \;\; G_{ij} :$$

$$\left\langle e^{2\Omega}(\rho + p_3)\right\rangle_A \propto \;(\rho^{4d} + p^{4d}) - \frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2 \; + \; \text{neg. semi-def.}$$

$$1 + w_{total} \geq 0 \qquad \leq 0 \qquad \leq 0$$

$$\left\langle e^{2\Omega}(\rho + p_k)\right\rangle_A \propto \;\frac{1}{2}(\rho^{4d} + 3p^{4d}) + \frac{k+2}{2k}\frac{1}{a^3}\frac{d}{dt}\left(a^3\langle\xi\rangle_A\right) \; + \; \begin{array}{c}\text{neg. semi-def.}\\ \text{for range of A}\end{array}$$

$$\boxed{1 + 3\,w_{total} \; < \; 0}$$

only hope
is if this is non-zero

$$\boxed{\leq \; 0}$$

Illustrative example: pure cosmological constant (w_{total} = -1)

$$\left\langle e^{2\Omega}(\rho + p_3)\right\rangle_A \propto (\rho^{4d} + p^{4d}) - \frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2 \;+\; \text{neg. semi-def.}$$

$$1 + \cancel{w_{total}} \geq 0 \qquad \leq 0 \qquad \leq 0$$

$$= 0$$

UH-oh

$$\left\langle e^{2\Omega}(\rho + p_k)\right\rangle_A \propto \frac{1}{2}(\rho^{4d} + 3p^{4d}) + \frac{k+2}{2k}\frac{1}{a^3}\frac{d}{dt}\left(a^3\langle\xi\rangle_A\right) \;+\; \begin{array}{c}\text{neg. semi-def.}\\ \text{for range of A}\end{array}$$

$$1 + 3\,w_{total} = -2$$

only hope

$$\leq 0$$

Theorem I:

Suppose dark energy described by theory obtained by:

1) compactifying from higher dimensions with 13>D>5

2) where higher D theory described by Einstein GR

3) where extra D is conformally Ricci-flat and

$$ds^2 = e^{2\Omega} g_{\mu\nu} dx^\mu dx^\nu + e^{2\Omega} g_{ab} dy^a dy^b$$

4) and extra D is closed or orbi/orientifolded

5) null energy condition (NEC) is satisfied

. . . then w_{total} must be > -1

PROVEN

Curious corollary:

Not only rules out pure Λ universe,
but also ΛCDM

That is, if Theorem I assumptions are correct,
then $w_{DE} > -1$

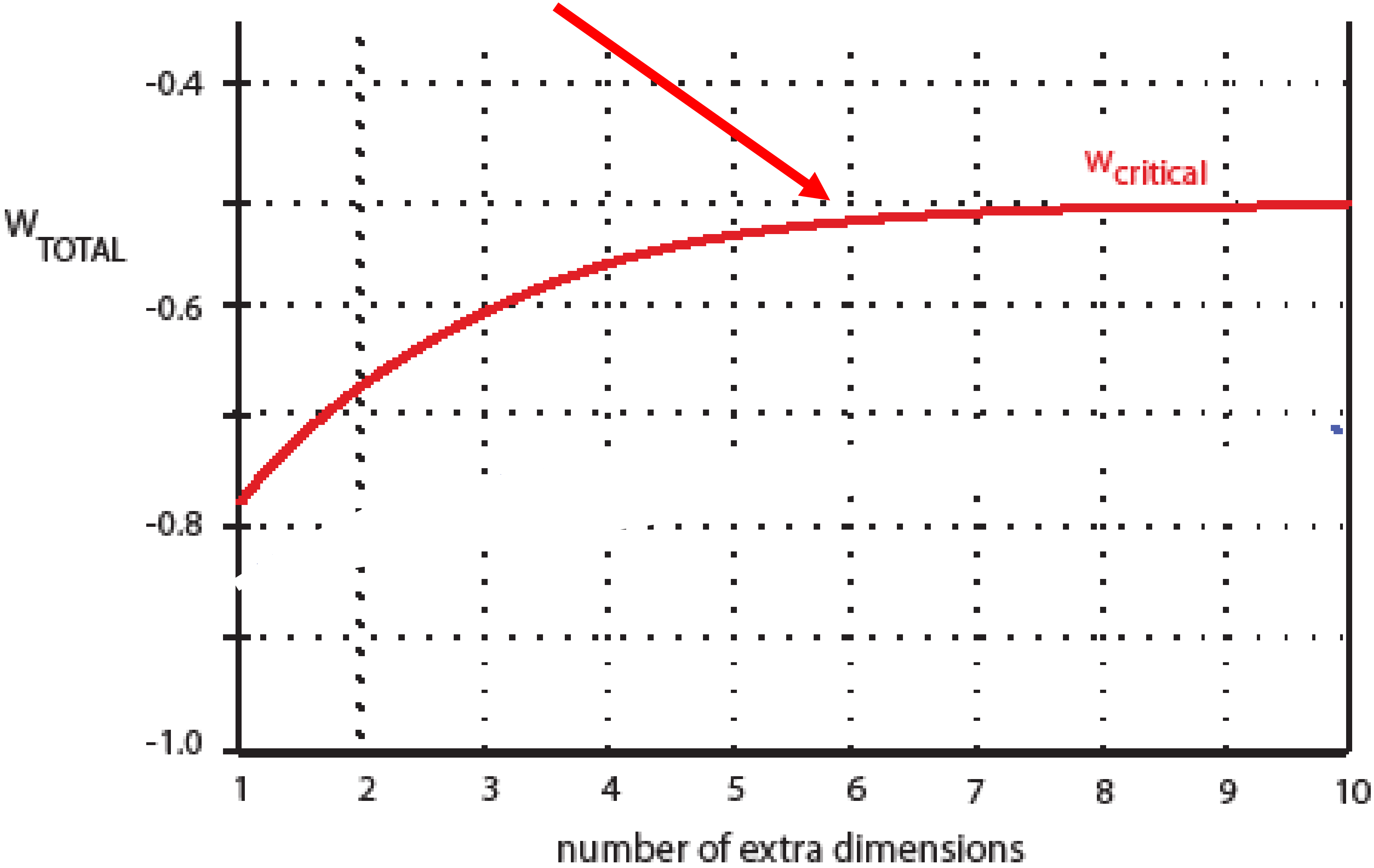
Are the Thm I conditions therefore ruled out observationally?

Theorem II: if all conditions of Thm I satisfied and

$$-1/3 > w_{total} > w_{critical} > -1$$

. . . can maintain acceleration indefinitely
(but G_N must vary with time to maintain the NEC)

requires $w_{total} > -0.53$ -- ruled out !



Are the Thm I conditions therefore ruled out observationally?

Theorem III: if satisfy all conditions of Thm I and

$$w_{critical} > w_{total} > -1$$

can maintain $w < w_{critical}$ for only a brief period; (w_{DE} and G_N must be rapidly time-varying)

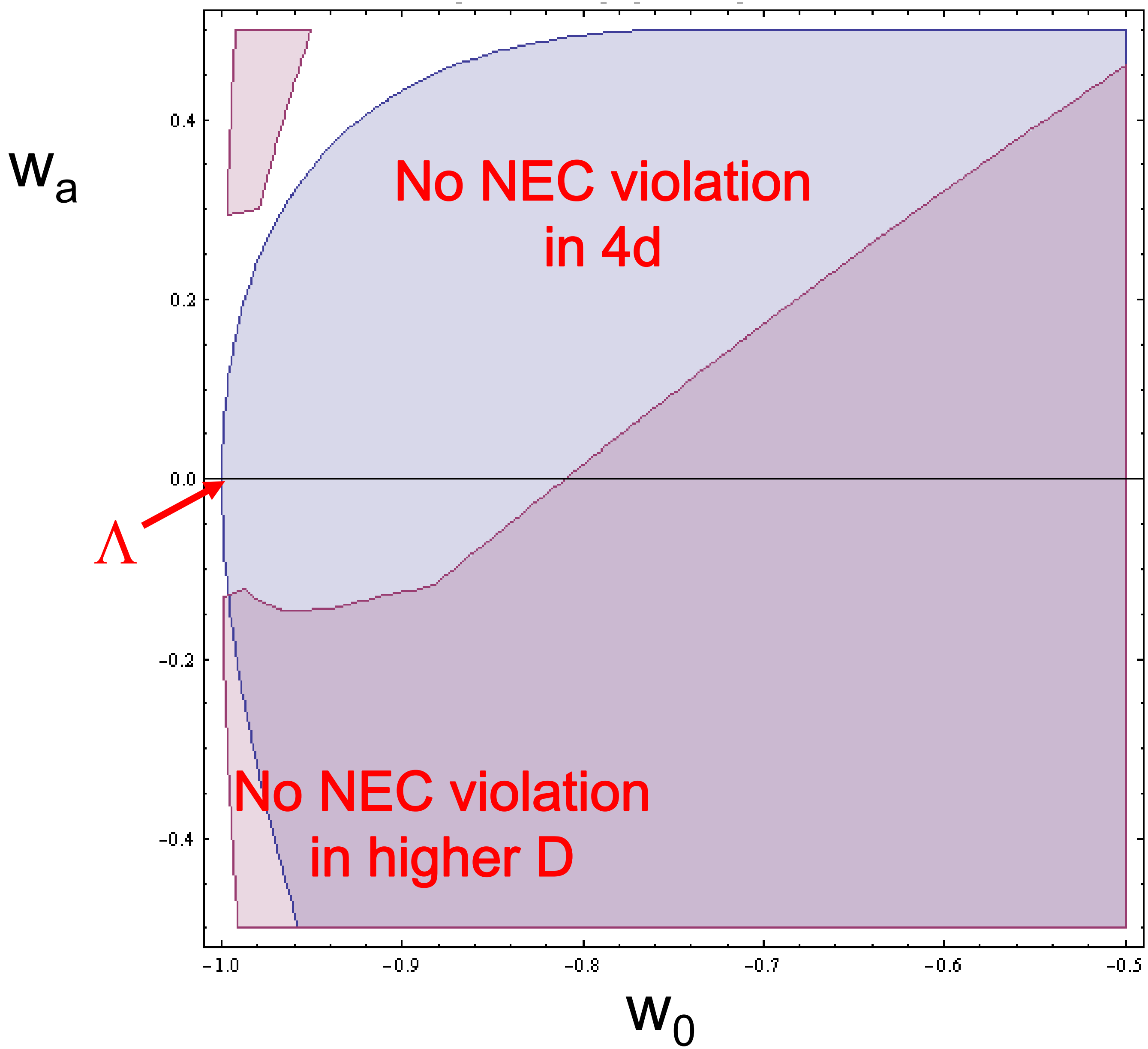
Practical example: $w_{total} = \Omega_{DE}\, w_{DE} = -0.73$ today

G_N: compare w/ model-independent limits on $\xi = \frac{H_0 \dot{G}}{G}$ & $\frac{|G_{BBN} - G_0|}{G}$

$$|\, d\, w_{total} / d\, z \,| < 1.3$$

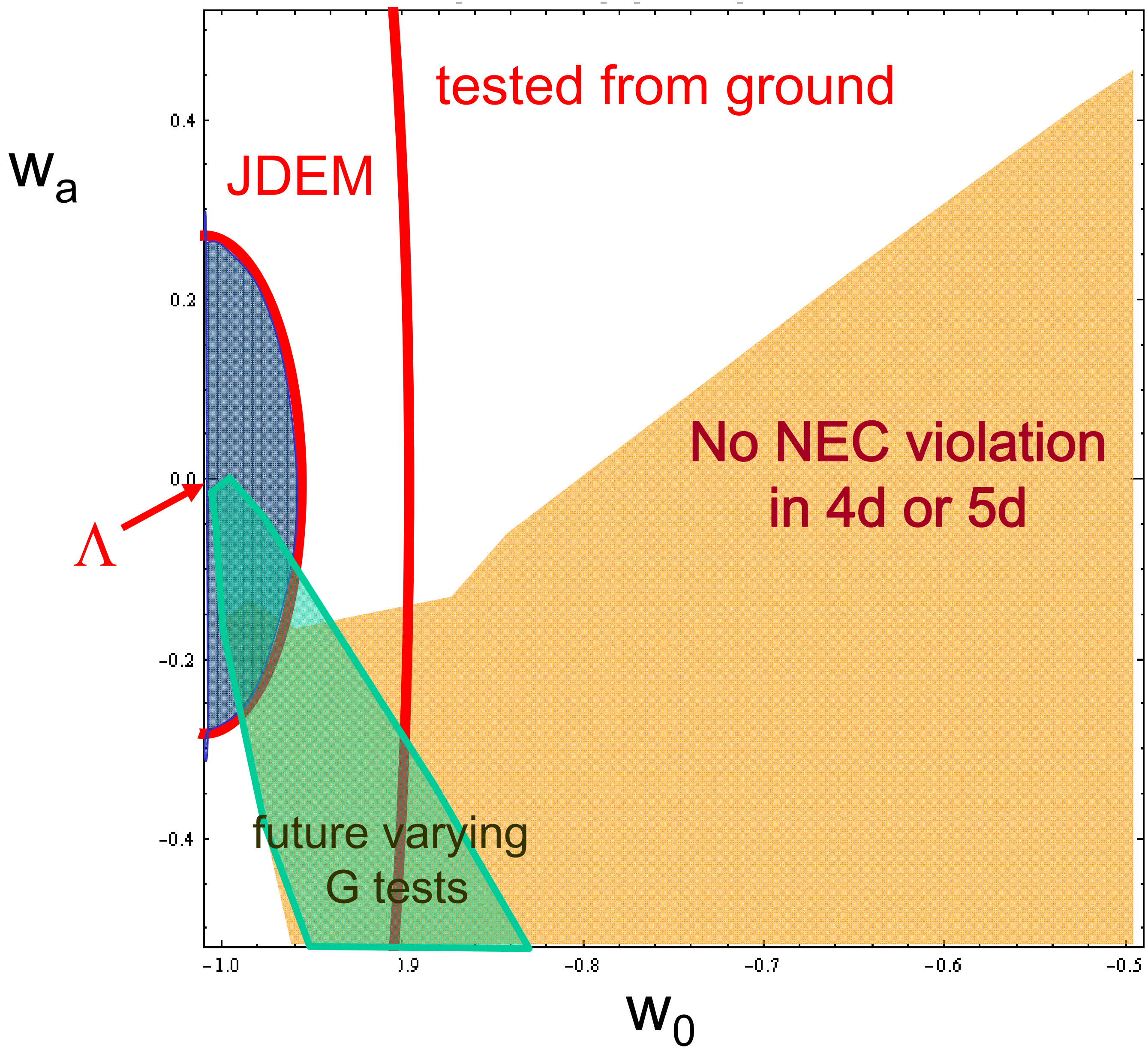
Models that satisfy constraints on $G_N(t)$ and NEC

$$w(a) = w_0 + w_a(1-a) + w_b(1-a)^2$$



Models that satisfy constraints on $G_N(t)$ and NEC

$$w(a) = w_0 + w_a(1-a) + w_{aa}(1-a)^2$$



preliminary

Are the Thm I conditions therefore ruled out observationally?

No, not yet,
although the situation could change
with forthcoming experiments

JDEM (Joint Dark Energy Mission) → JEDM (Joint Extra Dimensions Mission)

On the other hand,
Thm I conditions are *completely incompatible* with inflation !

Curious Corollaries:

- inflation requires violating at least one of the Thm I conditions in the early Universe
- Thm I provides interesting counterexample to principle underlying chaotic/eternal inflation

Theorem IV: If fundamental physics is obtained by::

1) compactifying from higher dimensions with 13>D>5

2) where higher D theory described by Einstein GR

3) where extra D is conformally Ricci-flat and

$$ds^2 = e^{-2\Omega}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2\Omega}g_{ab}dy^a dy^b$$

4) and extra D is closed or orbi/orientifolded

5) null energy condition (NEC) is <u>NOT</u> satisfied in higher D

6) G_N fixed (or very slowly varying)

. . . then, there is an A* such that NEC is violated for <u>any</u> w_{total} < -1/3 & violation must be in $\langle \rho + p_k \rangle_A$

Theorem V: If fundamental physics is obtained by::

1) compactifying from higher dimensions with 13>D>5

2) where higher D theory described by Einstein GR

3) where extra D is conformally Ricci-flat and

$$ds^2 = e^{-2\Omega}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2\Omega}g_{ab}dy^a dy^b$$

4) and extra D is closed or orbi/orientifolded

5) null energy condition (NEC) is <u>NOT</u> satisfied in higher D

6) G_N fixed (or very slowly varying)

. . . then NEC must be satisfied for same A*
for <u>all</u> $w_{total} > -1/3$

Curious Corollary:

Constraints on avoiding large G_N variation and
on the nature of NEC violation:

$$p_k{}^j = \tfrac{1}{2}\left(- \rho_{4d}{}^j + 3\, p_{4d}{}^j \right) = -\tfrac{1}{2}\, \rho_{4d}{}^j \,(1\text{-}3\, w_{4d})$$

radiation	$p_k{}^r = 0$
matter	$p_k{}^m = -\tfrac{1}{2}\, \rho_{4d}{}^m$
Λ	$p_k{}^\Lambda = -2\, \rho_{4d}{}^\Lambda$

But maybe not what you might expect !
e.g. can't be usual Λ or orientifold or Casimir energy alone !

Curious Corollary:

Constraints on avoiding G_N violation and
the kind of NEC violation:

$$p_k{}^j = ½ (- \rho_{4d}{}^j + 3\, p_{4d}{}^j) = - ½\ \rho_{4d}{}^j\ (1-3\, w_{4d})$$

radiation	$p_k{}^r = 0$
matter	$p_k{}^m = - ½\, \rho_{4d}{}^m$
Λ	$p_k{}^\Lambda = - 2\, \rho_{4d}{}^\Lambda$

… also not what you get just by having
scalar fields (inflatons) with flat potentials in 4d

Inflation more problematic than because you must sustain w close to -1 for 60 e-folds: hard to avoid violating NEC by a huge amount

$$\left\langle e^{2\Omega}(\rho+p_3)\right\rangle_A \propto (\rho^{4d}+p^{4d}) - \frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2 \quad + \quad \text{neg. semi-def.}$$

$$\left\langle e^{2\Omega}(\rho+p_k)\right\rangle_A \propto \frac{1}{2}(\rho^{4d}+3p^{4d}) + \frac{k+2}{2k}\frac{1}{a^3}\frac{d}{dt}\left(a^3\langle\xi\rangle_A\right) \quad + \quad \text{neg. semi-def. for range of A}$$

violation of NEC 10^{100} x DE → source of NEC different from DE & must be able to annihilate it more to come . . . .