

Math 480 – Topics in Modern Math

Prerequisites: A year of analysis at the 300 level or above (for example, Mathematics 360-361, or 508-509); a semester of linear algebra at the 300 level or above (for example, Mathematics 370).

Mathematics 480 will open with a review of the basics of real analysis (brief or extended as class background requires). The review will include: introduction of the real numbers through Dedekind cuts, continuity of real-valued functions on the “real line”; Cantor nested-interval principle, basic results for continuous functions, Maximum and Intermediate Value theorems, Heine-Borel Theorem, Uniform Continuity on closed intervals; metric spaces, convergence of sequences, Cauchy sequences, completeness, more general uniform continuity and intermediate value theorems; general topology, separation, compactness, product spaces, Tychonoff’s Theorem; special topics in analysis: Weierstrass Polynomial Approximation Theorem, Bernstein polynomials and simultaneous approximation of functions and derivatives, topics from “divergent series,” summation methods; rudiments of Lebesgue measure theory, the Lebesgue integral, L_p spaces, Hölder, Minkowski, and Cauchy-Schwarz inequalities; basics of Functional Analysis, normed spaces, Banach spaces and Hilbert space, with examples (L_p spaces, continuous-functions spaces), Banach spaces and spectral theory, groups and Fourier transforms, Tauberian theorems; approximation theory, again, through the prism of functional analysis; extension of the polynomial approximation theorem (Stone-Weierstrass theorem), Muntz approximation theorem (by polynomials with preassigned powers), compact operators, the Spectral theorem, Stone’s theorem (representations of the additive group of real numbers); Peter-Weyl theory (representations of compact groups). A selection from these topics as time and class preparation allow.