

What is number?  
A course Math 480/550, Spring 2008  
Tu-Th 12-1:30pm

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This is a short description of my Math 480/550 course in Spring 2008. The main source is my book “What is number?” published (in Russian) by “Nauka” in 1993 in the series “Modern mathematics for students”. The content of the book:

Introduction

Chapter 1. The chain

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O}$$

§ 1. From  $\mathbb{N}$  to  $\mathbb{Z}$  and from  $\mathbb{Z}$  to  $\mathbb{Q}$ : a Grothendieck group, Lie skew fields and derived categories

§ 2. From  $\mathbb{Q}$  to  $\mathbb{R}$ : the idea of completion; p-adic numbers and adels

§ 3. From  $\mathbb{Q}$  to  $\mathbb{R}$ : the idea of order; non-standard analysis

§ 4. From  $\mathbb{R}$  to  $\mathbb{C}$ ,  $\mathbb{H}$ ,  $\mathbb{O}$ : Clifford algebras, Dirac equation and the projective plane over the field  $\mathbb{F}_2$

Chapter 2. Other variants of numbers

§ 5. Matrices as numbers

§ 6. Continuous matrices and von Neumann factors

§ 7. What is supersymmetry?

§ 8. Discrete (lattice) differential and integral calculus,  $q$ -analogues

Bibliography

From the introduction:

When a school student first meet mathematics, (s)he is told that it is a science which studies numbers and figures. Later, in a college, (s)he learns analytic geometry which express geometric notions using numbers. So, it seems that numbers is the only object of study in mathematics.

True, if you open a modern mathematical journal and try to read any article, it is very probable that you will see no numbers at all. Instead, authors speak about sets, functions, operators, groups, manifolds, categories, etc.

Nevertheless, all these notions in one way or another are based on numbers and the final result of any mathematical theory usually is expressed by a number.

So, I think it is useful to discuss with math major students the question posed in the title. I want to show, what meaning can have the term “number” in modern mathematics, speak of some problems arising in this connection and of their solutuons.

I hope, this will help novices to orient themselves in the reach, beautiful and complicated world of mathematics.