# Math 104 Makeup Final Exam — Spring 2011

1. Find the area of the region bounded by the graphs of $y = x^2 + 5x$ and of $y = -x^2 + 3x$.

(a) 1/12 (b) 1/16 (c) 1/24 (d) 1/8 (e) 1/6 (f) 1/3

2. Consider the region in the $x, y$-plane that lies between the graphs of $x = 1$ and $y = 3 - x$, between $y = 0$ and $y = 1$. Find the volume of the solid obtained by rotating this region about the $y$-axis.

(a) $16\pi/3$ (b) $11\pi/2$ (c) $(\pi + 3)/4$ (d) $\pi^2 + 2$ (e) $17/\pi$ (f) $113/8$

3. Find the average value of the function $f(x) = (\ln x)/x$ on the interval $1 \leq x \leq 3$.

(a) $(\ln 6)/5$ (b) $(\ln 3)/6$ (c) $(\ln 2)/2$ (d) $((\ln 3)/2)^2$ (e) $((\ln 2)/3)^2$ (f) $(\ln(\ln 2))/2$

4. Evaluate $\displaystyle\int_0^{\pi/2} \cos(x) e^{\sin(x)}\, dx$.

(a) 0 (b) 1 (c) $e$ (d) $e + 1$ (e) $e - 1$ (f) $e^2 - e$

5. Evaluate $\displaystyle\int_2^4 x\,(\ln x)\, dx$.

(a) 0 (b) $e^4 - e^2$ (c) $14 \ln 2 - 3$ (d) $6 \ln 2 + \pi$ (e) $e\pi + 4 \ln 2$ (f) $\cos(4) - \cos(2)$

6. Evaluate $\displaystyle\int_0^{1/2} \sqrt{1 - x^2}\, dx$.

(a) $\dfrac{1}{\pi} + \dfrac{1}{4}$ (b) $\dfrac{\sqrt{2}}{3} + \dfrac{2}{7}$ (c) $\dfrac{2}{e} + \dfrac{\ln 2}{6}$ (d) $\dfrac{7}{32} - \dfrac{1}{2\pi}$ (e) $\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{8}$ (f) $\dfrac{\cos(1/2)}{2} - \dfrac{3}{\sqrt{\pi}}$

7. Evaluate $\displaystyle\int \frac{2}{y^3 + y}\, dy$.

(a) $2 \ln(y^3 + y) + C$ (b) $\ln\left(\dfrac{y^2}{y^2 + 1}\right) + C$ (c) $\dfrac{1}{y^2 + \ln y} + C$

(d) $\dfrac{1}{(y^3 + y)^2} + C$ (e) $\ln(2y^2 + 1) + C$ (f) $2 \ln |y| - y + C$

8. Evaluate the improper integral $\displaystyle\int_0^{\infty} \frac{x^2}{(x^3 + 1)^3}\, dx$.

(a) $\dfrac{1}{6}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{4}$ (d) $\sqrt{\pi}$ (e) $\dfrac{\sqrt{2}}{2^{27}}$ (f) It is divergent.

9. The curve defined by $y = \sqrt{1 + 2x}$, between $x = 1$ and $x = 3$, is rotated about $x$-axis. Find the resulting surface area.

(a) $\ln(2) + \dfrac{128\pi}{5}$ (b) $\dfrac{\pi}{3} + e^2$ (c) $\dfrac{6}{\pi} \cos^{-1}\left(\dfrac{1}{3}\right)$

(d) $\dfrac{4 + 20\,\pi}{\sqrt{3}}$ (e) $\dfrac{\pi + 1}{\sqrt{3}}$ (f) $\dfrac{16\,\pi}{3}(2\sqrt{2} - 1)$

10. Consider the region in the first quadrant that lies below the graph of $y = \sin(x^2) + \cos(x^2)$, to the left of $x = \sqrt{\pi/2}$. Let $A$ be the area of this region. Then the $x$-coordinate of the center of mass of this region is

(a) $\dfrac{1}{A}$ (b) $\dfrac{2}{A}$ (c) $A\sqrt{2}$ (d) $\dfrac{\pi}{3}$ (e) $\dfrac{\pi}{2A}$ (f) $\dfrac{A}{\sqrt{\pi}-1}$

11. Suppose that a certain probability density function $f(x)$ is given by:

$$f(x) = \frac{Ax}{(1+x^2)^2} \text{ for } x \geq 0; \quad f(x) = 0 \text{ for } x \leq 0,$$

for some constant $A$. Then the probability that the corresponding random variable lies between $-1$ and 1 is:

(a) $1/e$ (b) $1/\pi$ (c) $1/2$ (d) $2/3$ (e) $\pi$ (f) $1/\sqrt{2}$

12. Determine the limit of the sequence $x_n = \dfrac{(-3.14)^n}{\pi^n}$ as $n \to \infty$.

(a) 0 (b) $\ln 2$ (c) $1/\pi$ (d) $\sqrt{2}-1$ (e) $\ln(\pi/2)$ (f) The limit does not exist.

13. The series $1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \dfrac{1}{5^2} - \dfrac{1}{6^2} + \cdots$

(a) converges to a sum between 1/2 and 3/4.
(b) converges to a sum between 3/4 and 1.
(c) converges to a sum between 1 and 2.
(d) converges to a sum that is greater than 2.
(e) converges to a negative sum.
(f) diverges.

14. The series $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n\ln(n)}$

(a) converges by comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n}$. (b) diverges by comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n}$.
(c) converges by the ratio test. (d) diverges by the ratio test.
(e) converges by the integral test. (f) diverges by the integral test.

15. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^n}$

(a) converges by the $p$-test.
(b) diverges by the $p$-test.
(c) converges because the terms approach zero.
(d) diverges because the terms do not approach zero.
(e) converges by the alternating series test.
(f) diverges by the alternating series test.

16. Find the radius of convergence of the series $\sum_{n=0}^{\infty} n^2(n-3)^2(ex-\pi)^n$.

(a) $e$ (b) $1/e$ (c) $\pi$ (d) $1/\pi$ (e) $e/\pi$ (f) $\pi/e$

17. Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5 (f) The series diverges.

18. Consider the polynomial $1 - \frac{x^2}{2!}$ as an approximation to $\cos(x)$ on the closed interval $-1 \leq x \leq 1$. What is the best bound on the error that is given by Taylor's inequality?

(a) 1/36 (b) 1/24 (c) 1/10 (d) 1/2 (e) $\pi/2$ (f) $\cos(1)$

19. What is the general solution of the differential equation $\frac{dy}{dx} = -\frac{xy}{y^2+1}$? What is the behavior of the solutions as $x \to \infty$?

(a) $y = -\frac{1}{4}x^2 \ln(y^2+1) + C$ and $y \to -\infty$. (b) $y = -\frac{1}{4}x^2 \ln(y^2+1) + C$ and $y \to 0$.

(c) $\frac{y^2}{2} + \ln(y) = -\frac{x^2}{2} + C$ and $y \to 0$. (d) $\frac{y^2}{2} + \ln(y) = -\frac{x^2}{2} + C$ and $y \to \infty$.

(e) $\frac{y^3}{3} + y = -\frac{x^2 y}{2} + C$ and $y \to \infty$. (f) $\frac{y^3}{3} + y = -\frac{x^2 y}{2} + C$ and $y \to -\infty$.

20. Which of the following curves is an orthogonal trajectory to the family of curves $\frac{x^2}{4} + \frac{y^2}{9} = C$, where $C$ is an arbitrary positive constant?

(a) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (b) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (c) $y = x^{-4/9}$

(d) $y = x^{-9/4}$ (e) $y = 2x^{4/9}$ (f) $y = 2x^{9/4}$