University of Pennsylvania
Math 103 Fall 2008
Final Exam

NAME_____

Student ID #_____

Circle the name of your professor

RIMMER **WYLIE** **ZYWINA**

Work all problems in the space provided. You may use the back of each sheet for additional space. Please indicate when you do so.

There are 12 multiple choice questions each worth 5 points (no partial credit will be given).

There are 4 free-response partial credit questions each worth 10 points.

Please write legibly so that the proper credit may be given. You must show all work, an unsupported answer will receive little or no credit.

Transcribe your answers to this cover sheet. Do not tear this sheet off.

Problem	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Pts.	

Problem	Answer	Points
13		
14		
15 (a)		
15 (b)		
15 (c)		
16 (a)		
16 (b)		
16 (c)		
16 (d)		
16 (e)		
	Total Pts.	

Final Exam Score:

1. What is the slope of the tangent line to $f(x) = x\cos(x^2)$ at $x = \sqrt{\frac{\pi}{2}}$

A) $-\pi$ B) π C) 0 D) 1 E) -1 F) $\frac{1}{2}$

2. The function $f(x) = (x-3)^{2/3}$ is increasing for what values of x?

A) $(-\infty, \infty)$ B) $(3, \infty)$ C) nowhere

D) $(-\infty, 3)$ E) $(0, \infty)$ F) everywhere except 3

3. Find the value of c (if any) that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{1+x}$ on the interval $[0,1]$.

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{\sqrt{2}}{2}$ D) $2-\sqrt{2}$ E) $\sqrt{2}-1$ F) no values

4. What is the equation of the tangent line to the curve $x^3 + 2y^2 + 3xy = 6$ at the point $(1,1)$?

A) $y = \frac{-6}{5}x + \frac{11}{5}$ B) $y = -3x + 4$ C) $y = -2x + 3$

D) $y = \frac{-6}{7}x + \frac{13}{7}$ E) $y = \frac{-5}{6}x + \frac{13}{5}$ F) $y = -4x + 1$

5. Find the value of the limit.

$$\lim_{x \to 2} \frac{\sqrt{x+7} - 3}{(x-2)(x+1)}$$

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{9}$ D) $\frac{1}{27}$ E) $\frac{1}{18}$ F) Does not exist

6. The function $f(x) = x^4 - 6x^3$ is concave down for what values of x?

A) $(-\infty, \infty)$ B) $(3, \infty)$ C) $(0, \infty)$ D) $(-\infty, 0)$ E) $(0, 3)$ F) $(-\infty, 3)$

7. Evaluate.

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

A) 0 B) 1 C) 2 D) 3 E) 4 F) 5

8. Let $f(x)=\ln\left(3x^2+1+e^{-x}\right)$. Find $f'(0)$.

A) -1 B) 0 C) $\frac{1}{2}$ D) $-\frac{1}{2}$ E) e^{-1} F) $\sqrt{e}$

9. Find the value of the limit.

$$\lim_{x\to 0}\frac{e^x-1-x-\frac{x^2}{2}}{x^3}$$

A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) $\frac{1}{6}$ F) ∞

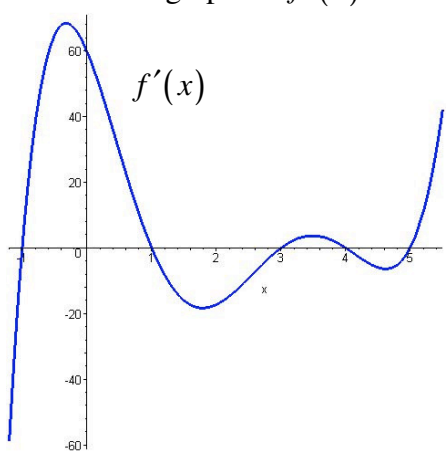
10. If the function $f(x)=\begin{cases}3ax^2+2bx+1, & x\le 1\\ ax^4-4bx^2-3x, & x>1\end{cases}$ is differentiable for all real values of x, then $b=$

A) $\frac{-11}{4}$ B) $\frac{1}{4}$ C) $\frac{-7}{16}$ D) 0 E) $\frac{-1}{4}$ F) $\frac{11}{4}$

11. Consider the function $f(x)=x^2e^{-x^2}$. The absolute maximum of $f(x)$ is:

A) e B) 1 C) e^{-1} D) e^{-2} E) e^{-3} F) e^{-4}

12. Consider the graph of $f'(x)$ below:



Let $a=$ the number of local maximums of $f(x)$.

$b=$ the number of inflection points of $f(x)$.

Find $a+b$.

A) 5 B) 4 C) 9 D) 7 E) 6 F) 8

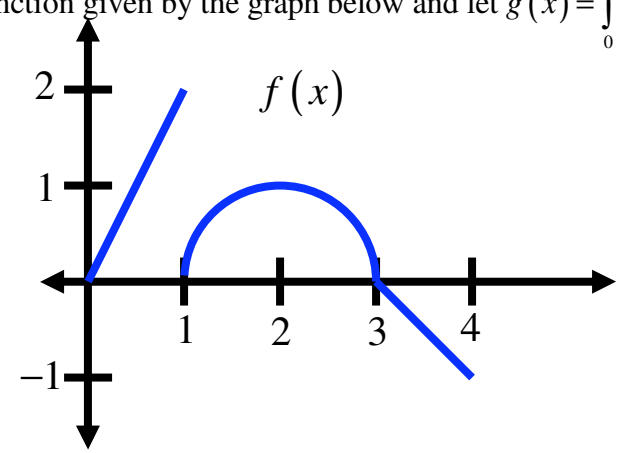
13. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 3ft/sec., how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

14. A theater currently sells tickets at a price of \$8, and sells 1000 tickets per show. It is estimated that for each 10-cent decrease in the price, an additional 20 tickets will be sold. What price should the theater charge to maximize its revenue?

15. For $0 \le t \le 5$, a particle moves in a horizontal line with acceleration $a(t) = 2t - 4$ and initial velocity $v(0) = 3$.

(a) When is the particle moving to the left?
(b) When is the particle speeding up?
(c) What is the position of the particle at time t if the initial position of the particle is 6?

16. Let $f(x)$ be the function given by the graph below and let $g(x) = \int_0^x f(t)\,dt$.



Find the following:

(a) $\lim_{x \to 1^-} f(x)$

(b) $f'(2)$

(c) $g'\left(\frac{1}{2}\right)$

(d) $g(4)$

(e) the value of x at which $g(x)$ attains its maximum