MATH 130 PROBLEMS

Linear, Quadratic and Exponential functions

The first true test of any scientific theory is whether or not people can use it to make accurate predictions. Calculus, being the study of quantities that change, provides the language and the mathematical tools to discuss and understand change in a precise, quantitative way. An important prerequisite to using calculus to analyze "real-world" situations is having a good understanding of the basic "elementary" functions (polynomials, exponentials, logarithms, trigonometric functions and all their compositions, inverses, etc.).

Linear functions:

The simplest, and probably the most familiar, kind of formulas and graphs to consider are the linear ones. As everybody knows, if the variable y depends linearly on x then y=mx+b, where m and b are parameters that specify the slope and y-intercept of the graph of the line, respectively. To get a little experience with Maple while reviewing a little about lines, solve the following problems:

1. The standard price for a car to make a round trip across the Benjamin Franklin Bridge (which connects Philadelphia and scenic Camden, NJ) is $3.00. But for commuters who live in southern New Jersey and work in Philadelphia (or vice versa), it is possible to buy a special sticker which costs $30.00. With the sticker, each round trip (including the first) costs an additional $2.00.

(a) Write down the expression which gives the total cost of making x round trips across the bridge at the regular rate. In Maple, define the variable r to be equal to this expression. (Don't forget to use the asterisk for multiplication, the := for assignment, and the semi-colon!). (b) Write down the expression which gives the total cost of making x round trips across the bridge for a commuter with a sticker. In Maple, define the variable c to be equal to this expression.

(c) Use Maple to plot r and c for x varying from 0 to 30 on the same set of axes. The easiest way to do this is to type

>plot({r,c},x=O..30);

Describe in words what you see and what it means for the typical commuter.

(d) There is a value of x (not an integer) for which *r* and c are equal. You can **find** it with Maple using the statement

> solve (r=c,x);

(e) So, how many trips does a commuter have to make in order for it to be worth her while to buy a sticker? Given that a sticker is valid for one month, should the typical commuter buy one? Why or why not?

2. (a) Do an analysis similar to that of the previous problem for the following situation: You are about to buy your first (used) car. You have a choice of buying a 1996 Chevy for $3650 or a 1996 Toyota for $4400. Your mechanic has assured you that both cars are in good shape and should give you many repair-free miles (you are quite a lucky consumer!). The Chevy gets 18 miles per gallon of gasoline and the Toyota gets 30 miles per gallon. Assume that gasoline costs $1.90 per gallon. How many miles would you have to drive before the cost of buying and fueling the Chevy becomes greater than the cost of buying and fueling the Toyota?

(b) Make up your own "linear" problem. Write it up and give it to another student to solve. You should also solve the other student's problem. In your homework write-up, you should include only the statement of the *other* student's problem and your solution of that problem. Be sure to include the other student's *name.*

3. You know that the equation of a line is *y=mx+b.* To get some practice applying Maple to understand how varying the parameters of a family of curves affects the curves, we will do it for lines.

(a) In Maple, it is possible to define a list of expressions. Define *lineist* to be a list of lines with the same slope but different y-intercepts as follows:

> linelst:= 3*x-5, 3*x-4 ,3*x-3, 3*x-2, 3*x-1, 3*x, 3*x+1, 3*x+2, 3*x+3, 3*x+4, 3*x+5;

You can plot all of these expressions on the same set of axes using a statement like:

> plot({1inelst},x=-3..3);

This should illustrate clearly the effect of changing *b* in the equation of a line (but you already knew that, right?).

(b) Generate and plot a list of expressions which illustrate the effect of changing the m in *mx+b.*

Quadratic functions

After linear functions and graphs, quadratic ones are the next simplest. A quadratic function is one of the form y:=:a*xr'2+b*x+c for some fixed values of a, b, and c.

4. Before we go any farther, generate and graph three lists of quadratic functions (as you did in the previous problem) which illustrate the effects of changing a, b, and c in a quadratic

expression. DO NOT TURN IN ALL OF THE GRAPHS!! Write down in words what you observe about the various effects.

Quadratic functions are especially important in physics problems which involve the motion of a body which undergoes a constant acceleration. The most common example of such a body is one which is falling (near the surface of the earth) under the influence of gravity (and neglecting air resistance). In this case, the magnitude of the downward acceleration is 32 feet per second per second.

In physics or calculus class, the following formula is derived for the position of a body undergoing a constant acceleration of magnitude a for t seconds:

> pos:= 0.5*a*t^2 + v0*t +x0;

In this expression, a is the constant acceleration, x0 is the initial position coordinate and v0 is the initial velocity of the body.

5. Here is an interesting example: In 1979, one of the first magnetically levitated trains was tested in Japan. The mag-lev train is suspended and propelled by magnetic forces. The nature of these forces provides the tram with a constant acceleration and the same magnitude constant deceleration. So, throughout the first half of a typical maglev journey the tram is accelerating and throughout the second half the tram is decelerating. The test train traversed a straight 7000-meter-long track, starting from rest, and it reached a maximum speed (at the midpoint of the journey) of 144 meters per second. The problem is to find the value of a, the constant acceleration, and also how long the trip took.

To do this, consider the first half of the trip. The position of the train at the halfway point is governed by the quadratic equation given above. But we know that x0=0 and v0=0. We also know that pos=3500 at the end of the first half of the trip. There is another equation we can write, concerning the velocity at any time, it says that

> vel:=a*t+v0;
Given the data in our problem, you can define the two Maple expressions and

> solve({pos=3500, vel=144});

Try it, and explain your results. You may need to use evalf(...) to get a good idea of what Maple's initial answer means.

Exponential functions

It may be the case that exponential functions arise even more frequently in nature than linear ones. The exponential function is by far the most important function in all of mathematics (this

becomes more and more apparent as one considers generalizations of exponential functions from real to complex numbers and beyond to more exotic situations). For the moment, we will stick to real-valued functions.

Pure exponential functions are written (in Maple) as y=a*b^x, where a and b are given constants, and b must be positive. Examples would be y=3*6^x or y=5*(2/3)^(1/x).

6. Use Maple to plot several exponential functions on the same set of coordinate axes. Write a few sentences describing the roles of a and b. Try to use words that have some correspondence with the ideas of slope and y-intercept of lines.

You should have discovered that exponential curves all have basically the same shape–for values of b bigger than 1 and positive values of a, the value of y rises slowly at first (for negative x) but then begins to rise more and more rapidly, until finally it is shooting almost straight up. For values of b less than 1 and a positive, the value of y decreases -- quickly at first and then more and more gradually, approaching but never quite reaching zero. So when b>l, the exponential function is an increasing function and the equation y=a*bx is called an equation of exponential growth. When 0 < b < 1, the phenomenon being modeled is one of exponential decay.

In an exponential growth situation, the rate at which the quantity y is increasing is proportional to the value of y -- in other words, the more y you have, the faster you get more. This is the case (for example) with populations of animals or bacteria – the larger the population, the more organisms there are to produce still more, so the population increases faster, etc.. Another situation which produces exponential growth is an exothermic (i.e., heat-producing) chemical reaction: the more of the reaction that is happening, the more heat is produced, which in turn causes the reaction to proceed faster, which in turn produces more heat, etc. etc..

For an example of exponential decay, consider the process by which kerosene is purified to make jet fuel (see p. 16 of the text). Pollutants are removed by passing the kerosene through a special clay filter. Suppose the clay is in a pipe and that each foot of the pipe removes 20% of the pollutants that enter it. Therefore, each foot leaves 80% of the pollutants. If P0 is the initial amount of pollutant, and P=f(n) is the quantity left after n feet of pipe, then f(1)=0.8*P0, f(2)=(0.8)*(0.8)*P0, etc.. and one reasons that P:=P0*(0. 8)^n.

7. Assume P0=1 and make a table of and plot P for various n using Maple. How many feet of pipe does it take before only half the pollutants remain? You can estimate this from your table and graph, but how do you find out exactly (e.g., is there an equation you can solve? with Maple?). How many feet before only 10% of the pollutants remain?

8. In 1980, there were about 170 million vehicles (cars and trucks) and about 227 million people in the United States. If the number of vehicles has been growing at about 4% per year, while the number of people has been growing at 1% per year, in what year was there, on the average, one vehicle per person?

Make sure you plot the two relevant exponential curves, you will probably need fsolve to determine exactly where they cross. BE CAREFUL to state clearly what variables you are using and what they mean (time scales, etc..).

Benjamin Franklin's will:

The Franklin Institute of Philadelphia and the Franklin Technical Institute of Boston owe their existence in part to a provision in Benjamin Franklin's will:

"...I wish to be useful even after my Death, if possible, in forming and advancing other young men that may be serviceable to their Country both in Boston and Philadelphia. To this end I devote Two thousand Pounds Sterling, which I give, one thousand thereof to the Inhabitants of the Town of Boston in Massachusetts, and the other thousand to the Inhabitants of the City of Philadelphia, in Truset and for the Uses, Interests and Purposes herinafter mentioned and declared ..."

Franklin's plan for the money given to Boston was to lend money to young apprentices at 5% interest with the provision that each borrower should pay each year

"... with the yearly Interest, one tenth part of the Principal, which sums of Principal and Interest shall be again let to fresh Borrowers If this plan is executed and succeeds as projected without interruption for one hundred Years, the Sum will be one hundred and thirty-one thousand Pounds of which I would have the Managers of the Donation to the Inhabitants of the Town of Boston, then lay out at their discretion one hundred thousand Pounds in Public Works. The remaining thirty-one thousand pounds, I would have continued to be let out on Interest in the manner above directed for another hundred Years ....At the end of this second term if no unfortunate accident has prevented the operation the sum will be Four Millions and Sixty-one Thousand Pounds ...."

Unfortunately, it was not always possible to find as many borrowers as Franklin had planned, but the Bostonian managers of the trust did the best they could. In January 1894, at the end of 100 years from the inception of the Franklin gift, the fund had grown from 1000 pounds to almost 90,000 pounds. In 100 years the original capital had multiplied about 90 times instead of the 131 times Franklin had imagined.

9. (a) Was Franklin right? If 1000 pounds were invested for 100 years at 5% interest, would the balance be 131,000 pounds? You get to adjust the frequency of compounding. Explore this graphically and numerically with Maple.

(b) What rate of interest, compounded annually, would have multiplied Franklin's original bequest by 90? How about compounded continuously?

Using "solve" and "fsolve" in Maple:

Maple has two powerful commands for solving equations: The solve command can solve algebraic equations involving many variables and parameters -- it always attempts to find exact solutions to problems. The fsolve command performs numerical calculations and tries to provide good approximations to the answers to problems.

When you use solve you give the equation (or equations) you would like to solve, followed by the variable (or variables) you would like to solve for. For example, solve the equation

> eqn:= Q=QO*a^(n*t);

$$eqn := Q = QO \, a^{(nt)}$$

for t. Notice that we give equations names so we can refer to the whole equation later. To solve this problem in Maple, we type:

> solve (eqn, t)

$$\frac{\ln\left(\dfrac{Q}{QO}\right)}{\ln(a) \, n}$$

This is the answer you would have gotten by hand.

10. Use the solve command to answer the following problem:
The half-life of a certain radioactive substance is 12 days. If there are 10.32 grams of the substance initially: (a) write an expression to determine the amount of substance, A, as a function of time; (b) find the amount of time required to reduce the amount of substance to 1 gram. Be sure to include a sentence that explains what your variables stand for, and what the answer means.

Sometimes when we use solve, we get answers that we wouldn't have expected. For example, to find all real roots of the equation:

> eqn2 :=x^4=20;

$$eqn2 := x^4 = 20$$

we do the following:

> solve (eqn2,x);

$$20^{(1/4)}, I20^{(1/4)}, -20^{(1/4)}, -I20^{(1/4)}$$

What a mess! There are four answers, and it helps to give them names:

> solns:=solve(eqn2,x):
> solns[1];

$$20^{(1/4)}$$

> solns[2];

$$I20^{(1/4)}$$

> solns[3];

$$-20^{(1/4)}$$

> solns[4];

$$-I20^{(1/4)}$$

Maple has the habit of giving exact answers when it can, but $20^{(1/4)}$ is not too helpful, so we can get more meaningful values using evalf:

>evalf(solns[1]);

$$2.114742527$$

The I in solns [2] and solns [4] stands for $\sqrt{-1}$ which is usually written with a lower case i, but Maple starts all names of constants with a capital letter. There are two complex solutions to the equation. So sometimes Maple gives us more than we ask for.

On the other hand, we can use the time-honored technique of plotting, then solving numerically (i.e., using fsolve):

> plot(x^4-20,x=-3..3);
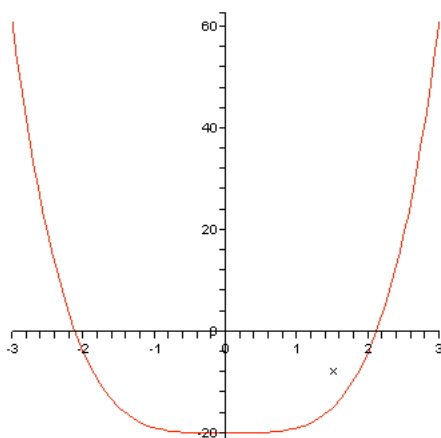
We can see the two roots, and to use fsolve to find them we use:

> fsolve(eqn2,x);

$$-2.114742527, \ 2.114742527$$

Sometimes fsolve fails to find all the roots, or there are infinitely many, as in

> fsolve(cos(x)=0.3,x);

$$1.266103673$$

If we want a solution in a different range, we can specify it as in:

> fsolve(cos(x)=0.3,x,x=3..6);
$$5.017081634$$

11. Find all (real) roots of sin(t)=t^2. Use a plot and explain why you know how many roots there are.

12. In photography, the relationship between the constant focal length of a lens (F), the distance between the object being photographed and the lens (u) and the distance between the lens and the place where the image will be in focus (v) (you want this to be where the film is) is given by:

> 1/u+1/v=1/F;

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{F}$$

(a) Use Maple to solve this equation for v in terms of F and u.

(b) Now assume that F is 50 mm. This makes v a function of u in the answer to part (a). Plot this function v vs. f(u) as u ranges over the interval [50mm,50000mm].

(c) "Depth of field" refers to the range of distances in which an object remains in focus (on the film). Use the graph you produced in part (b) to explain why the depth of field is broad when you focus on an object very far from the lens but is narrow for objects close to the lens. If you have learned about derivatives, use the language of derivatives to explain your answer.

Trigonometric functions --

13. The approximate number of hours of daylight at about 35 degrees north latitude (e.g., somewhere in North Carolina) on the 21st of each month is given in the following table:

Month:	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Sun:	10.08	11.06	12.18	13.34	14.22	14.65	14.20	13.33	12.20	11.04	10.09	9.68

Find (and plot) a sine curve (i.e., a function of the form A*sin(B*t+C) which predicts the hours of sunlight for any given day of the year for this location. Be sure to explain the units of t in your function and the meaning of the constants A, B and C.

Problems about Derivatives

You can use Maple to compute derivatives of expressions and functions. For example, to take the derivative of the expression x^2-sin(x) in Maple, you can enter:

> diff(x^2-sin(x) ,x);

$$2 x - \cos(x)$$

This last expression gives the slope of the graph of x^2-sin(x) for various values of x. It will be easier if we name things:

> y:=x^2-sin(x)

$$y:=x^2 - \sin(x)$$

> dy:=diff(y,x);

$$dy :=2x - \cos(x)$$

The meaning of diff(y,x) is "take the derivative of y with respect to x". To plug numbers into the expressions y and dy, we use the "subs" command of Maple:
> subs(x=1,y);

$$1 - \sin(1)$$

> evalf(%);

$$.1585290152$$

> subs(x=l,dy);
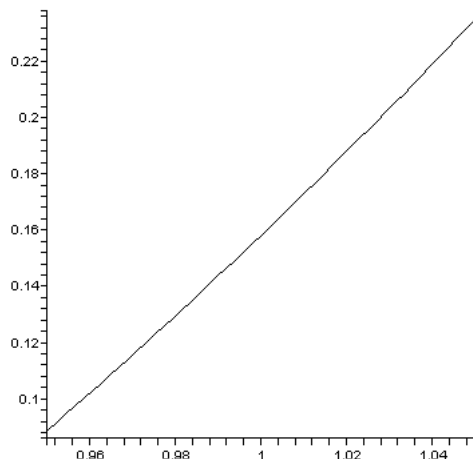
$$2 -\cos(1)$$

> evalf(%);

$$1.459697694$$

From these statements and their output, we conclude that the graph of y=x^2-sin(x) goes through the point (1,0.1585..), and as it does, the slope of the graph is 1.459... , i.e., it is increasing fairly rapidly. To verify this, we plot the graph of y on a short domain containing 1:

> plot (y,x=0.95. .1.05,color=black);

1. (a) Calculate the values of $y=x^3 - 4x + \cos(x)$ and of its derivative at x=1. Interpret these as values and slopes of the graph of y -- illustrate your conclusions with a plot.

(b) Choose another expression, and use Maple to take its derivative - then repeat part (a) for two different points on the graph of your expression.

Maple has another way of computing derivatives. This second way deals with Maple functions, whereas the first way dealt with Maple expressions. Functions are special because they are defined using the "arrow" notation:

> f:=x->x^3-sin(x);

$$x^3 - \sin(x)$$

> df:=D(f);

$$df := x \rightarrow 3x^2 - \cos(x)$$

The result of the D operation is also a function. One advantage of functions is that we can plug values into them without using "subs" -- we use ordinary math notation:

> f(1);

$$1 - \sin(1)$$

> df(1);
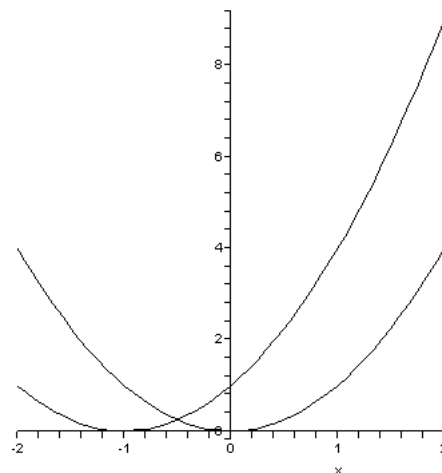
$$3 - \cos(1)$$

> evalf(df(L));

$$2.459697694$$

Using function notation: Here is a plot of f(x) and of f(x+ 1) for f(x)=x^2 on the same axes for x=-2..2:

> f:=x->x^2: > plot ({f(x),f(x+1)},x=-2..2);

2. (a) Describe anything of interest you note about this graph. (b) Plot f(x-1) and f(x) on the same axes for x=-2..2 then describe anything of interest you note. (c) Now plot f(x+h) and f(x) for h=0.3 on the same axes for x=-2..2. Try it for smaller h's. What happens to the plots as h closes in on 0?

Average growth rates:

In class (see the demonstration ml03dm6), we used the program "aroc" to compute average rates of change for functions on the way to calculating their derivatives.

> aroc:=(f,x,h) ->(f(x+h) -f(x)) /h;

$$aroc:= \frac{f(x+h)-f(x)}{h}$$

To use aroc to calculate the average rate of change of f(x)=x^2 on the interval x=3..4, we say (since 4=3+1, h=1)
>f:=x->x^2:
> aroc(f,3,1);

$$7$$

Taking the derivative involves figuring out what happens as h tends to zero:

> aroc(f,3, 0.5) ,aroc(f,3,O. 1) ,aroc(f,3,0 .01) ,aroc(f,3,O. 0001);

$$6.500000000, 6.100000000, 6.010000000, 6.000100000$$

It looks as though the derivative of f at 3 is 6:

> D(f)(3);

$$6$$

3. (a) Find the average growth rate of the function g(x)=ln(0. 5+x^4) on the interval from 3 to 6.
(b) Ditto, on the interval from 3 to 4 .... 3 to 3.1 .... 3 to 3.01..
(c) What do you think the derivative of ln(0.5+x^4) is at x=3? Check your guess using Maple.

4. For a small enough value of h, aroc(f,x,h) is almost the same as D(f)(x). For a function f of your own choosing, plot both of aroc(f,x,0.05) and D(f)(x) on the interval x=-1 ..1. How far apart are the graphs? Why are they almost the same? Why are they different?

You know that for values of x where the derivative of a function is positive, then the function increases. And where the derivative is negative the function decreases. So at a high or low point on a graph, the derivative should be zero.

5. (a) How does this observation suggest a strategy for finding the high and low points on a graph of a function? In Maple? (Your strategy should involve the commands diff or D and solve or fsolve).

(b) Use your strategy from part (a) to find the highest point on the graph of $x^{10}e^{-x}$ for $x > 0$. Use a plot to support your answer.

6. Begin with x=11.37. Use the derivative to explain what happens to the value of $\ln((3*x^2+x+2)/(x^2+1))$ as x increases a little bit. Support your answer with a plot.

7. Here is the graph of a fairly exotic function, and of its derivative -- explain which is which:

> ex:=exp(-x^2/5)*cos(ln(1+x^4)): plot({ex,diff(ex,x)},x-3..3);
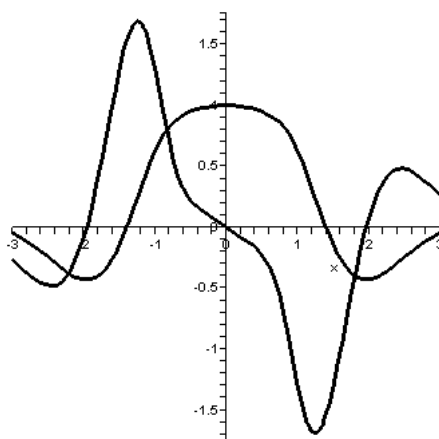
8. Graph f(x)=sin(x) and g(x)=sin(2x) from x=0 to x=2π, also graph f '(x) and g'(x). Be sure to describe and explain the similarities and differences in the graphs of the derivatives.

9. The population of a herd of deer varies seasonally and can be modeled by

$$P(t) = 4000 + 400\sin\left(\frac{\pi}{6}t\right) + 180\sin\left(\frac{\pi}{3}t\right)$$

where t is measured in months from the first of April.
a) plot the graph of this function and its derivative over a period of one year showing how the herd size varies with time;
b) when is the herd largest? how many deer at that time?
c) when is the herd smallest? how many deer at that time?
d) when is the herd growing the fastest? when is the herd shrinking the fastest?
e) how fast is the herd growing on April 1?

Problems about Integrals

Maple has several commands for illustrating integrals, and one basic command for computing them. These have been used in the demonstrations, the use of the int command is described in the front of this manual, as well as in the Maple Learning Guide.

Illustrating Reimann Sums

In Maple's "student" library, there are several commands that can be used to illustrate the computation of integrals via Riemann sums. To use these commands, you must first enter the command:

> with (student);
[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope, summand, trapezoid]

When you use a semicolon, you will get a list of all of the commands available in the student library. Use a colon at the end of the line to surpress the output

We demonstrate the use of the commands by working through example 1 on p. 230 of the text: There, you are to compute:

> Int(1/t,t=1..2);

$$\int_1^2 \frac{1}{t} dt$$

The "Int" command with a capital I is used to get Maple just to type integrals. It does not evaluate them -- the "int" command with a small i does that. But the form of the two int (for integral) commands is the usual "verb(what,how);" form of many Maple commands. Before the comma, you enter the (name of the) expression to be integrated ("Integrate what?") and after the comma you give the name of the variable and the range of integration ("Integrate it how?"). We will illustrate only the left-hand sums.

Maple has three relevant commands: "leftbox", which draws pictures, "leftsum", which gives summation notation for the area under the left boxes, and "value", which computes the sums (evalf can also be used). So, as we approximate the above integral using 10 rectangles which touch the curve on the left, we can have Maple illustrate the boxes by typing:

> leftbox(1/t,t=1..2,10);
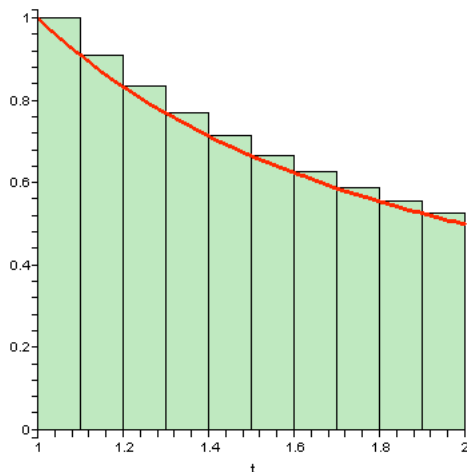
Notice that the sum of the areas of the boxes gives an overestimate of the area under the curve because the curve is concave up.

To calculate the sum of the areas of the boxes, use the command

> leftsum(1/t,t=1..2,10);

$$\frac{1}{10}\sum_{i=0}^{9}\frac{1}{1+\frac{1}{10}i}$$

In this expression, Maple has numbered the ten boxes from 0 to 9, and the area of the ith box is equal to 1/10*(1/(1+iIl 0)). Notice that 1/10 is delta-t and the other factor is the value of the function I /t at the left side of the box. To calculate the sum, we can ask Maple for:

> value(%);

$$\frac{33464927}{46558512}$$

> evalf(%);

.7187714032

This is the value given near the end of the example. Maple is capable of calculating (but not drawing) the area for an arbitrary number of boxes:

> leftsum(1/t,t=1..2,n);

$$\frac{1}{n}\sum_{i=0}^{n-1}\frac{1}{1+\frac{1}{n}i}$$

> value(%);

$$\frac{n\Psi(2n)-n\Psi(n)}{n}$$

The "Psi" function is a special function used by number theorists. The point of getting Maple to do this is we can now ask Maple what happens as the number of boxes approaches infinity:

> limit (%,n=infinity)

$$\ln(2)$$

And this is in fact the value of the integral, as we can check using Maple's int command (which uses the Fundamental Theorem of Calculus to evaluate integrals):

> int(1/t,t=1..2);

$$\ln(2)$$

There are commands "rightbox" and "rightsum" that correspond to "leftbox" and "leftsurn".

Here are some problems for you to try:

1.   Use Maple to do problems 12, 13 and 14 on page 235 using left and right sums. When you do this, have Maple draw the pictures for 2, 10 and 50 subdivisions (the picture for 250 subdivisions doesn't look so good, but you might want to try it anyway—it does show rather graphically what is going on as the number of subdivisions increases). For each of these problems, then have Maple compute the integrals using int.

Sometimes, int fails to produce an answer (because there is no way to write down the function whose derivative is the integrand). In these cases, using evaf on the integral will give a (very good) approximation of the area under the curve. For instance:

> int(x^x,x=1..2);

$$\int_{1}^{2}x^{x}dx$$

When int (small i) just prints out the integral, it means Maple cannot figure out how to use the Fundamental Theorem to solve the problem – usually (but not always), this means that the problem cannot be done using the Fundamental Theorem. In cases like this, to get the area under the curve, we can:

> evalf(%);

2.050446235

2.   Use this method to do any of the integrals that Maple got stuck on in the preceding problem.

3.   One of the recurring themes in the text's problems involves jumping out of a plane with a defective parachute…so here we go again.
If you jump out of an airplane and your parachute fails to open, your downward velocity t seconds after you jump is approximated pretty well by

$$v(t) = \frac{g}{k}\left(1 - e^{-kt}\right)$$

where: $g = 9.8$ m/sec^2 and $k = 0.2$ sec^{-1}.
a) write an expression for the distance you fall in T seconds; and
b) if you jump from an altitude of 5000 m above the ground, estimate, using left and right sums, how many seconds you fall before hitting the ground.

If you do part (a) correctly, your answer for d(t), the distance you fall in t seconds, should involve the integral of the function v(t) given in the text. Now use Maple to:

(i) Make plots of the functions v(t) and d(t).
(ii) Use fsolve to figure out exactly when you hit the ground.
(iii) Calculate your average velocity during the entire free-fall.

4.   If $F(x) = e^{x^2}$, it can be shown that $f(x) = F'(x) = 2xe^{x^2}$. Find $\int_0^1 2xe^{x^2}\,dx$ two ways:

a) using left and right sums; and,
b) using the Fundamental Theorem of the Calculus.
Illustrate and compute the Riemann sum (with 25 left or right boxes). You can use int and evalf to do part b).

5.   Assume that the population of Mexico is modeled by
$$P(t) = 67.38(1.026)^t$$
where t is the number of years since 1980.
a) What is the average population of Mexico between 1980 and 2000?
b) What is the average of the population in 1980 and the population in 2000?
c) Explain, in terms of the concavity of the graph of P, why your answer to part b) is different from your answer to part a).

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