Math 104 Final Exam, May 1, 2003 Your Name: ____________________

Your TA: ____________________

PART I

Circle your answers.

1.	(A)	(B)	(C)	(D)	(E)	5.	(A)	(B)	(C)	(D)	(E)
2.	(A)	(B)	(C)	(D)	(E)	6.	(A)	(B)	(C)	(D)	(E)
3.	(A)	(B)	(C)	(D)	(E)	7.	(A)	(B)	(C)	(D)	(E)
4.	(A)	(B)	(C)	(D)	(E)	8.	(A)	(B)	(C)	(D)	(E)
9.	(A)	(B)	(C)			10.	(A)	(B)	(C)		

PART II

1. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^n(2n+1)}$ is: ____________________

Show your work in the space below.

2. $\int_0^{1/2} \frac{1}{(1+5x^2)^{3/2}}\, dx = \frac{\square}{\square}$. Show your work on the back of this sheet.

This examination consists of two sections. All work to be graded must be written on the answer sheet. At the conclusion of the exam, please hand in *only* the answer sheet. If you require extra sheets of paper to show your written work then please make sure your name is written on each piece of paper that you hand in.

The first part consists of 10 multiple-choice questions, worth 8 points each. Please circle the letter (A - E, or for the last two series questions, A - C) that corresponds to your choice. There is no partial credit and you do not need to show your work. If you select more than one choice or make no selection, then you will automatically receive no credit for the problem.

The second part consists of 2 written-answer problems. The first problem, worth 10 points with a possibility of partial credit, asks you to find the interval of convergence of a power series. The second is an integration problem, for which you should state your answer (2 points) and show your work (8 points, with partial credit for progress towards the solution).

No calculators or computers may be used during this exam. No books may be used. You are permitted one 8 1/2" by 11" sheet of notes, and beyond this, no notes may be used.

PART I: Multiple-choice questions

Select one answer (A - E) on the answer sheet, for each question (8 points each).

1. What is the value of $\int_0^\pi \sqrt{1+\cos\theta}\, d\theta$?

A. 1 B. $\frac{\pi}{2}$ C. π D. 2 E. $2\sqrt{2}$

2. $\lim_{t\to 0} \frac{t\sin t}{1-\cos t} = \ ?$

A. 0 B. 1 C. 2 D. -1 E. -2

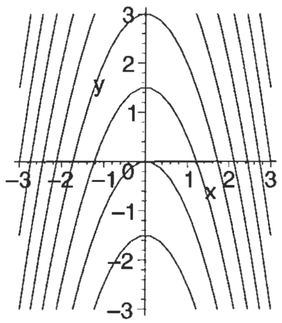
3. The area between the graph of $y=(e-1)x$ and $y=e^x-1$ is rotated around the y-axis to produce a solid. What is the volume of the solid?

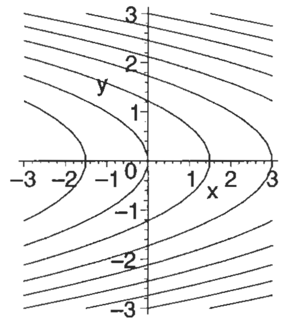
A. $\pi(\frac{23}{6}-\frac{11}{3}e+\frac{5}{6}e^2)$ B. $\frac{\pi}{3}(2e-5)$ C. $\pi(3-e)$
D. $2\pi(\frac{7}{3}e-\frac{11}{6})$ E. $2\pi(\frac{2}{3}e-\frac{9}{5})$

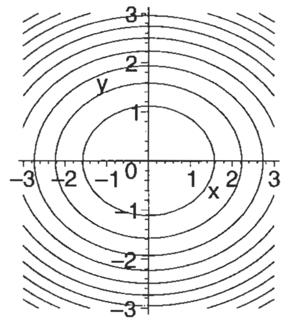
4. If $a_n = \left(\frac{2n+1}{2n-1}\right)^n$ then $\lim_{n\to\infty} a_n = \ ?$

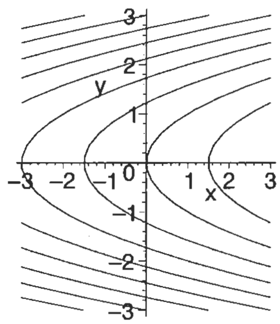
A. 2 B. e^2 C. e D. 1 E. 0

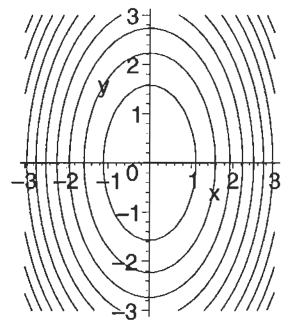
5. Let $z = f(x, y) = x^2 + y$. Then the level curves for z are:

A. 

B. 

C. 

D. 

E. 

6. Let $y = \left(1 - \frac{x}{2}\right)^{-1/2}$. The binomial series for y is

A. $1 + \frac{1}{4}x - \frac{1}{32}x^2 + \frac{1}{128}x^3 - \frac{5}{2048}x^4 + \cdots$

B. $1 - \frac{1}{4}x + \frac{3}{32}x^2 - \frac{5}{128}x^3 + \frac{35}{2048}x^4 - \cdots$

C. $1 + \frac{1}{4}x + \frac{3}{32}x^2 + \frac{5}{128}x^3 + \frac{35}{2048}x^4 + \cdots$

D. $1 + \frac{1}{4}x + \frac{1}{32}x^2 + \frac{1}{128}x^3 + \frac{5}{2048}x^4 + \cdots$

E. $1 - \frac{1}{4}x - \frac{1}{32}x^2 - \frac{1}{128}x^3 - \frac{5}{2048}x^4 - \cdots$

7. Let $z = f(x, y) = \tan^{-1}(y/x)$. Then $\dfrac{\partial^2 z}{\partial x\, \partial y} = ?$

A. $\dfrac{x^2 - y^2}{(x^2 + y^2)^2}$ B. $\dfrac{\tan^{-1}(y/x)}{\sec^2(y/x)}$ C. $\dfrac{y^2 - x^2}{(x^2 + y^2)^2}$

D. $\dfrac{1}{(x^2 + y^2)^2}$ E. $\dfrac{x^2 - y^2}{x^2 + y^2}$

8. Let $a_n = \frac{n^2}{2^n}$. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$. Let

$$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}.$$

Then

A. $L = \frac{e}{2}$ and the series therefore converges

B. $L = \frac{1}{2}$ and the series therefore converges

C. $L = \frac{e}{2}$ and the series therefore diverges

D. $L = \frac{1}{2}$ and the series therefore diverges

E. $L = 1$ and the Ratio Test is inconclusive

9. The series $\sum_{n=1}^{\infty} \frac{1-n}{1+n}$

A. converges absolutely B. converges conditionally C. diverges

10. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

A. converges absolutely B. converges conditionally C. diverges

PART II: Written-answer problems

1. (10 points) Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^n(2n+1)}.$$

Please write your answer in the blank on the answer sheet and show your work on the answer sheet in the space indicated.

2. (2 points for correct answer; 8 points for work shown) Evaluate the following integral:

$$\int_0^{1/2} \frac{1}{(1+5x^2)^{3/2}}\,dx.$$

The answer is a positive rational number. Please write your answer (numerator over denominator) in the blanks provided. Show your work on the back of the answer sheet.