Math 114 **FINAL EXAM** **May 1, 2003**

1. Find a unit vector orthogonal to both of the vectors $< 1, -1, 0 >$ and $< 1, 2, 3 >$.

(a)$< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ (b) $< -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ (c) $< \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ (d) $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} >$

(e) $< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 >$ (f) $< -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 >$

2. If $f(x, y) = x^3y + e^{x+3y}$ compute $\frac{\partial^2 f}{\partial x \partial y}$ at $x = 1, y = 0$.

(a) e (b) 6 (c)$3 + 3e$ (d) $6 + e$ (e) $3 + e$ (f) None of the above.

3. Let S be the surface $x^2y + 4xz^3 - yz = 0$. An equation for the tangent plane to S at $(1, 2, -1)$ is

(a) $y + 5z = -3$ (b) $x - y + z = 0$ (c) $2x + y + 5z = 0$ (d) $x - 3z = 4$

(e) $2x - 3y + z = 3$ (f) $x + y + z = 2$

4. If $f(x, y) = x^4 - y^2 - 2x^2 + 2y - 7$, find the critical points and classify them as a relative maximum, a relative minimum, a saddle point or no conclusion possible.

(a) $(0, 1)$ relative maximum; $(-1, 1)$ saddle point
(b) $(0, 1)$ relative minimum; $(1, 1)$ relative minimum
(c) $(0, 1)$ no conclusion; $(-1, 1)$ relative maximum
(d) $(0, 1)$ relative minimum; $(-1, 1)$ and $(1, 1)$ saddle points
(e) $(0, 1)$ saddle point; $(-1, 1)$ no conclusion
(f) $(0, 1)$ saddle point; $(-1, 1)$ and $(1, 1)$ relative maxima

5. An ant is placed on a flat plate whose temperature distribution is given by $T(x, y) = 3x^2 + 2xy$. If the ant's initial position is $(3, -6)$, it should walk in which direction to cool off most rapidly?

(a) $-3i + 6j$ (b) $3i - 6j$ (c) $-i - j$ (d) $4i - 3j$

(e) $-i + 12j$ (f) $i + j$

6. Let $z = x - y$ and let $x = 4(t^3 - 1)$, $y = \ln t$. Find $\frac{dz}{dt}$ for $t = 1$.

(a) 12 (b) 0 (c) 9/5 (d) 11 (e) 7 (f) 2

7. If y is the solution of the initial value problem $y' + 2xy = x$ subject to $y(0) = 1/2$, what is $y(1)$? .

(a) $\frac{1}{2} + 1/e$ (b) $\frac{1}{2}$ (c) $\frac{1}{2} - 1/e$ (d) $1/e$ (e) $\frac{1}{2} + 2/e$ (f) $-\frac{1}{2}$

8. Solve the initial value problem $y'' + 10y' + 29y = 0$, $y(0) = 0$, $y'(0) = 10$. Then $y(\frac{\pi}{2}) =$

(a) $5e^{-5}$ (b) $5e^{-5\pi/2}$ (c) $5e^{-5}\pi$ (d) $5e^{-5}\frac{\pi}{2}$ (e) 0 (f) None of the above

9. Let $A = (2, 1, 3), B = (1, 2, -2)$ and $C = (-1, 3, 1)$. Then the plane through A, B and C intersects the z-axis at:

(a) 3 (b) 13 (c) 8 (d) 32 (e) 0 (f) None of the above

10. The area of the surface formed by revolving the portion of the curve $x^2+y^2 = 1$ in the first quadrant about the y-axis, is equal to:

(a) 2π (b) π (c) $2\pi^2$ (d) $4\pi^2$ (e) 4π (f) None of the above

11. How far does a bug travel from time $t = 1$ to time $t = 2$ if at time t it is at the point (t^2, t^3).

12. Find the area inside the circle $r = 1$ but outside the figure eight $r^2 = \cos(2\theta)$.

13. The planes $5x + 3y + 2z = 0$ and $2x + 8y - 5z = 0$ intersect in a line. Give an equation for this line.

14. Find the linearization of $f(x, y) = e^{yx^2}$ at $(1, 1)$.

15. Evaluate the double integral $\iint_R e^{-x^2-y^2} dA$ where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 1$ and the coordinate axes.

16. Compute the triple integral $\iiint_R z dV$ where R be the solid region inside both the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z^2 = x^2 + y^2$.

17. Compute the volume underneath the graph of e^{y^2} and over the triangle with vertices $(0, 0), (0, 1)$ and $(2, 1)$.

18. Find all fourth roots of -1.

19. Solve $y'' + y = \sin(2x)$ with $y(0) = 1, y'(0) = 1/3$.

20. Solve $(y^3 - y^2 \sin x - x)dx + (3xy^2 + 2y \cos x)dy = 0$