

**MATHEMATICS 240
FINAL EXAMINATION**

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Answer all questions by circling the **entire** statement you deem correct in each question. No books, tables, notes, calculators, computers or cell phones allowed. You may bring one 8.5" x 11" sheet of paper bearing any handwritten material you deem necessary and you may use both sides of this paper. **No partial credit.** Use the backs of your exam pages for scratch work and calculations.

(Your name—please print)

(Your PENN ID number)

(Your signature)

(Your instructor's name (print))

PLEASE DO NOT WRITE BELOW THIS LINE

PROBLEM	SCORE	PROBLEM	SCORE	PROBLEM	SCORE
I		VI		XI	
II		VII		XII	
III		VIII		XIII	
IV		IX		XIV	
V		X		XV	

TOTAL-----

(Your name)

I) For the differential equation

$$y'' - x^2y' + xy = 0$$

and the solution, $y(x)$, determined by the initial conditions $y(0) = 0$ and $y'(0) = 1$, when we write $y(x)$ in a power series, the coefficient of x^4 in this series is:

- a) $-1/24$ b) $1/2$ c) $-1/6$ d) $2/3$ e) 0
-

II) Given the vectors: $(1, t, -1)$; $(0, 1, -2)$; $(1, (s + 2), -s)$, find the condition on t and s so that these vectors are linearly dependent.

- a) $s = 2t + 5$ b) $s = 2t - 5$ c) $t - s = 5$
d) $t + s = 5$ e) none of these
-

III) An iron bead is constrained to move along the x -axis and is acted on by a varying magnetic field. The differential equation describing its motion is:

$$x'' + (1/2)x' + 2x = \cos(t).$$

Suppose $x(0) = 4/5$ and $x'(0) = 2/5$, then $x(3\pi/2)$ equals:

- a) $4/5$ b) $3/5$ c) $-2/5$ d) $2/5$ e) $-4/5$
-

IV) Suppose A and B are 3×3 matrices and $\det A = x \neq 0$ while $\det B = y$. Let C be the matrix $(2A)^{-1}B$, then $\det C$ is:

- a) $y/8x$ b) $2xy$ c) $-2y/x$ d) $2y/x$ e) $8y/x$
-

 (Your name)

V) Given a 4 x 4 matrix, M, we form the augmented 4 x 8 matrix $Y = (M | I)$ and row reduce Y to row reduced echelon form. When this is done we get the new matrix

$$\tilde{Y} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If $X = (x_1, x_2, x_3, x_4)$ and $b = (b_1, b_2, b_3, b_4)$, where the x_i are unknowns and the b_i are given numbers and each of X and b is viewed as a column vector, then we consider the matrix equation $MX = b$ and for it we can say:

- a) For each b there is a unique solution, but it may not be Cb.
- b) There are vectors b for which there is no solution.
- c) For each given b there are always many solutions.
- d) For each b there is a unique solution; it is Cb.
- e) Need more information about M and b to decide among the above choices.

 VI) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ in which C is the curve $\mathbf{r}(t) = (t, t^2, t^3)$ for $0 \leq t \leq 1$ and \mathbf{F} is the vector field $(e^y, xe^y, (z + 1)e^z)$.

- a) 1/2 b) 2e c) 0 d) e e) e/2

 VII) Recall that for a matrix its trace is the sum of its eigenvalues and its determinant is the product of its eigenvalues. Which of the following is **false**?

- a) There is a symmetric 3x3 matrix with an eigenvalue 2, trace 5 and $\det = 4$.
- b) For any 3 x 3 matrix, A, the matrix $A(A^T)$ is always diagonalizable.
- c) Symmetric matrices always have an orthonormal basis of eigenvectors.
- d) There is a symmetric 3x3 matrix with an eigenvalue 2, trace 5 and $\det = 6$.
- e) None of the above.

 (Your name)

- VIII) Consider the region Ω whose boundary consists of **two** curves: $x^2 + y^2 = 4$ and $x^2 + y^2 = \frac{1}{2}$ oriented and labeled as shown in the sketch below. Write C for the boundary of Ω and let \mathbf{F} be the vector field

$$\mathbf{F}(x, y) = (-y/[(x-1)^2 + y^2], (x-1)/[(x-1)^2 + y^2]).$$

Then $\int_C \mathbf{F} \cdot d\mathbf{r}$ equals:

- a) 2π b) -4π c) 4π d) 0 e) -2π
-

- IX) When we solve the differential equation

$$x^2(2+x)y'' + 2xy' - 3y = 0$$

by the Frobenius method, the exponents, r , in the solution $y = x^r \sum c_i x^i$ are:

- a) $r = \pm 2$ b) $r = -1 \pm \sqrt{3}$ c) $r = \pm \sqrt{3}$ d) $r = \pm \sqrt{3/2}$ e) $r = \pm 1$
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- X) Evaluate the line integral $\int_C (y + e^{\sqrt{x}})dx + (2x - \cos(y^2))dy$, where C is the boundary of the region enclosed between the curves $y = x^2$ and $x = y^2$ and C is oriented counter-clockwise.

- a) $1/3$ b) 3 c) 0 d) $\pi/3$ e) π
-

(Your name)

XI) Suppose A is the 3 x 3 matrix:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 2 \end{pmatrix}$$

Then

- a) A has eigenvalues 1, 1, 2 and is not diagonalizable.
- b) A has eigenvalues 0, 1, 2 and is not diagonalizable.
- c) A has eigenvalues 1, 1, 2 and is diagonalizable.
- d) A has eigenvalues 0, 1, 2 and is diagonalizable.
- e) None of the above.

XII) For the initial value problem:

$$y'' - 3y' + 2y = U(t-1); \quad y(0) = 0, \quad y'(0) = 1,$$

in which U(t) is the function that is 0 when $t < 0$ and 1 when $t \geq 0$, the Laplace transform of y, namely $L(y)(s)$, is given by

- a) $1/(s^2 - 3s + 2) + e^{-s}/(s(s^2 - 3s + 2))$
 - b) $1/(s^2 - 3s + 2) - e^{-s}/(s(s^2 - 3s + 2))$
 - c) $1/(s^2 - 3s + 2)$
 - d) $1/s(s^2 - 3s + 2)$
 - e) Cannot be determined
-

 (Your name)

XIII) Consider the surface, S, formed by the upper half of the ellipsoid

$$x^2 + y^2 + 6z^2 = 1,$$

and write C for the circle $x^2 + y^2 = 1$ where S cuts the xy-plane. We use the outer normal (upward pointing) to orient S so that C is traversed counter-clockwise in the xy-plane (when viewed from above). Let \mathbf{F} be the vector field

$$(\sin(xz) + \sqrt{z}, (3 + z)x - e^y, x^2 - y^3 - z^5)$$

Compute the surface integral $\iint_S \mathbf{curl} \mathbf{F} \cdot \mathbf{n} \, dS$:

- a) $-\pi$ b) π c) -3π d) 3π e) none of these

 XIV) We solve the differential equation $y'' + (t - (t^3/6))y = 0$ by a series $\sum a_i t^i$ and assume the initial conditions $y(0) = y'(0) = 1$. Then the expression

$$(a_2)^3 + (a_3)^2 - a_4$$

equals:

- a) $-1/3$ b) $1/9$ c) $1/12$ d) $-1/12$ e) $1/6$

 XV) Write S for the part of the surface $z = x^2 + y^2$ over the disc $x^2 + y^2 \leq 1$ in the plane. We orient S so that its normal, \mathbf{n} , points downward. If the vector field, \mathbf{F} , is given by: $\mathbf{F}(x, y, z) = (e^y + x, e^{\sin(z)} + \sin(x), -z + xy)$, then the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$ equals:

- a) 2π b) 4π c) π d) $\pi/2$ e) None of these.

 (END OF THE EXAM)

