Math 240, MAKEUP FINAL EXAM
January 10, 2007

INSTRUCTIONS:

1. Please complete the information requested below and on the second page of this exam. There are 15 multiple choice problems. No partial credit will be given.

2. You must show all your work on the exam itself. Blind guessing will not be credited. An answer with no supporting work may receive no credit even if correct.

3. For each problem which you want graded, transfer your answer to the answer sheet *on the second page of the exam.* We will not grade problems for which answers have not been marked on the answer sheet. Do NOT detach the answer sheet from the rest of the exam.

4. You are allowed to use one hand-written sheet of paper with formulas. NO CALCULATORS, BOOKS OR OTHER AIDS ARE ALLOWED.

- **Name** (please print):
- **Name of your professor:**

  ◯ **Dr. Kadison** ◯ **Dr. Pantev** ◯ **Dr. Shatz**

- **Signature:**
- **Last** 4 **digits of student ID number:**
- **Recitation day and time:**

NAME (please print):

ANSWERS TO BE GRADED

1. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
2. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
3. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
4. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
5. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
6. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
7. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
8. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
9. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
10. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
11. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
12. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
13. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
14. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯
15. A ◯ B ◯ C ◯ D ◯ E ◯ F ◯

(1) Find all real values of c for which the matrix $A = \begin{pmatrix} 1-c & c \\ -c & 1+c \end{pmatrix}$ is diagonalizable as a real matrix?

(A) $c = \pm 1$ **(B)** $c = 0$ **(C)** $c \leq -1$
(D) $c \geq 1$ **(E)** $|c| > 1$ **(F)** none of the above

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(2) Consider a matrix A, a vector of unknowns $\overrightarrow{x}$, and a vector $\overrightarrow{b}$ given by

$$A = \begin{pmatrix} 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ -6 & 0 & -2 & -3 \end{pmatrix}, \qquad \overrightarrow{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \qquad \text{and} \quad \overrightarrow{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

Which of the following statements is ***true***?

(A) rank$(A) = 1$ and the solutions of the linear system $A\overrightarrow{x} = \overrightarrow{b}$ depend on 3 parameters.

(B) rank$(A) = 2$ and the solutions of the linear system $A\overrightarrow{x} = \overrightarrow{b}$ depend on 2 parameters.

(C) rank$(A) = 3$ and the solutions of the linear system $A\overrightarrow{x} = \overrightarrow{b}$ depend on 1 parameter.

(D) rank$(A) = 2$ and the linear system $A\overrightarrow{x} = \overrightarrow{b}$ has no solutions.

(E) rank$(A) = 1$ and the solutions of the linear system $A\overrightarrow{x} = \overrightarrow{b}$ depend on 4 parameters.

(F) rank$(A) = 3$ and the solutins of the homogeneous linear system $A\overrightarrow{x} = \overrightarrow{0}$ depend on 1 parameters.

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(3) Which of the following statements is ***false***:

(A) The span of the rows of a matrix A is the same as the span of the rows of the row reduced echelon form of A.

(B) If A is a 2×2 matrix for which $A\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}4\\8\end{pmatrix}$, then $A\begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}8\\16\end{pmatrix}$.

(C) If A is diagonalizable 3×3 matrix, then A^{-1} is also diagonalizable.

(D) If A is a real 3×3 matrix and $A^2 = A$, then either $A = 0$, or $A = I_3$.

(E) An $n \times n$ real matrix can have at most n linearly independent eigenvectors.

(F) If $\vec{v} = \begin{pmatrix}3\\0\\-4\end{pmatrix}$, then the matrix $A = \vec{v}\,\vec{v}^T$ has rank 1.

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(4) Let

$$A = \begin{pmatrix} 0 & 1 & 12 \\ 0 & 0 & 6 \\ 0 & 0 & 1 \end{pmatrix}$$

Find the sum of the entries in the third column of the matrix A^{11}.

(A) 6

(B) 19

(C) 0

(D) 12

(E) 2

(F) 8

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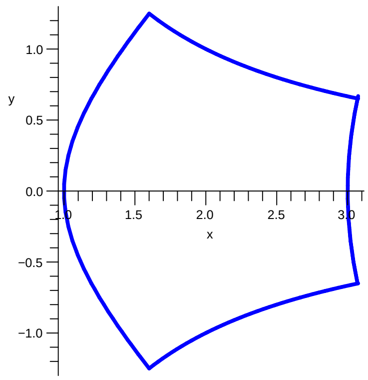
(5) Evaluate

$$\iint_R (x^2+y^2)\sin xy\, dA,$$

where R is the region in the xy-plane with $x \geq 0$ and bounded by the graphs of $x^2 - y^2 = 1$, $x^2 - y^2 = 9$, $xy = 2$ and $xy = -2$.



(**Hint:** use the change of variables given by $u = x^2 - y^2$, $v = xy$.)

(A) 16 **(B)** $\frac{1}{2}(1-\ln 2)$ **(C)** 0 **(D)** diverges **(E)** $\frac{1}{2}$ **(F)** $\frac{315}{4}$

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(6) Let

$$\overrightarrow{F} = (x + 2e^{yz})\,\widehat{\mathbf{\imath}} + x^3 z^4\,\widehat{\mathbf{\jmath}} + \left(\cos(y^2) - z^2\right)\,\widehat{\mathbf{k}},$$

and let S be the sphere of radius 1 centered at the origin. Evaluate $\iint_S \overrightarrow{F} \cdot \overrightarrow{n}\, dS$, where $\overrightarrow{n}$ is the outward unit normal.

(A) 125π **(B)** -25π **(C)** 0

(D) $-\dfrac{500\pi}{3}$ **(E)** $\dfrac{4\pi}{3}$ **(F)** none of the above

(7) Evaluate the line integral

$$\int_C -y dx + x dy,$$

about the curve C given by $\overrightarrow{r}(t) = a\cos(t)\,\widehat{\mathbf{\imath}} + b\sin(t)\,\widehat{\mathbf{\jmath}}$, awhere $a, b > 0$ and $0 \leq t \leq 2\pi$.

(A) $2\pi a^2 b^2$ **(B)** $2\pi b^2$ **(C)** 1

(D) $2\pi a^2$ **(E)** $2\pi ab$ **(F)** none of the above

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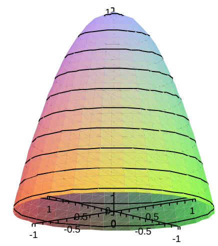
(8) Let S be the portion of the paraboloid $z = 1 - x^2 - y^2$ where $z \geq 0$ and let $\overrightarrow{F} = xyz\,\widehat{\mathbf{k}}$. Evaluate

$$\iint_S \operatorname{curl}(\overrightarrow{F}) \cdot \overrightarrow{n}\, dS,$$

where $\overrightarrow{n}$ is the unit normal vector to S that has a positive third component.



(A) 0 **(B)** -2π **(C)** $\dfrac{1}{6}$

(D) $-\dfrac{1}{6}$ **(E)** $-\pi$ **(F)** none of the above

(9) Let $y(x)$ be the solution of the equation $y'''+2y''-3y' = 0$, subject to the initial conditions $y(0) = 1$, $y'(0) = 1$, $y''(0) = -3$. Find $\lim_{x\to+\infty} y(x)$

(A) $2/3$ **(B)** -1 **(C)** $-\infty$
(D) does not exist **(E)** $4/3$ **(F)** none of the above

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(10) Let $y(x)$ be a solution of the initial value problem

$$2x^2y'' + 3xy' - 15y = 0, \quad y(1) = 1, y'(1) = -3.$$

What is $y(2)$?

(A) 0 **(B)** 1 **(C)** $-1/3$
(D) 4 **(E)** $1/8$ **(F)** none of the above

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(11) Let $F(s)$ be the Laplace transform of the function $f(t) = t + \sin(3t)$. Find $F(3)$

(A) 1/2 **(B)** 1/9 **(C)** 27

(D) 6 **(E)** 1/27 **(F)** -1

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(12) Let $Y(s)$ be the Laplace transform of the function $y(t)$. Find the Laplace transform of the spring-mass equation with damping and sharp impulse function,

$$y'' + 2y' + 4y = \delta(t-2), \quad y(0) = 1, \; y'(0) = -1$$

(A) $s^2 Y(s) - s - 1 + 2sY(s) + 4Y(s) = \mathcal{U}(t-2)$.

(B) $s^2 Y(s) - s - 1 + 2sY(s) + 4Y(s) = e^{-2s}$.

(C) $s^2 Y(s) + s - 3 + 2sY(s) + 4Y(s) = e^{-2s}$.

(D) $s^{-2} Y(s) + s^{-1} + 3 + 2s^{-1} Y(s) + Y(s) = s^{-1}$.

(E) $4Y(s) - 3s - s^2 = e^s$.

(F) $4sY(s) - 3s - s^2 = e^s$.

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(13) Find the first three non-zero terms of the power series solution about the ordinary point $x = 0$ of the initiial value problem

$$y'' + 2xy' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

(A) $x - \frac{2}{3}x^3 + \frac{4}{15}x^5$ **(B)** $1 - x^2 + \frac{1}{2}x^4$ **(C)** $x - \frac{x^3}{6} + \frac{x^5}{120}$

(D) $x + \frac{1}{3}x^3 + \frac{2}{5}x^5$ **(E)** $1 + x - x^2$ **(F)** none of the above

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(14) Consider the differential equation

$$2x^2y'' - xy' + (1+x)y = 0.$$

It has a Frobenius power series solution $y(x) = x^\alpha \sum_{j=0}^{\infty} a_j x^j$ for

(A) $\alpha = \pm\frac{1}{2}$ **(B)** $\alpha = \frac{1}{2}$ and $\alpha = 1$ **(C)** $\alpha = 0$ and $\alpha = \frac{\sqrt{5}}{2}$

(D) $\alpha = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ **(E)** $\alpha = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$ **(F)** $\alpha = \frac{1}{2}$ and $\alpha = \frac{\sqrt{5}}{2}$

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(15) Suppose that A is a real 2×2 matrix and that one of its eigenvalues is $-1+2i$ and that the corresponding eigenvector is $\begin{pmatrix} 1 \\ -i \end{pmatrix}$. Let $\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ be the real-valued solution of the initial value problem

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}, \qquad \mathbf{x}(\pi/2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

What is $x_1(0) + x_2(0)$?

(A) 0 **(B)** $\pi/2$

(C) $-e^{\pi/2}$ **(D)** 1

(E) -1 **(F)** none of the above
