

## MATH 141 Demonstration 12

### First-order differential equations

Maple has a basic function ("**dsolve**") for solving differential equations.

The important differential equations plotting programs are loaded into the system by typing:

```
> with(DEtools):
```

Let's start with an easy first-order equation, namely:

```
> eqn:=diff(y(x),x)-3*y(x)=x^2;
```

$$eqn := \left( \frac{\partial}{\partial x} y(x) \right) - 3 y(x) = x^2$$

Notice that we always indicate that y is a function of x by writing **y(x)**. The whole differential equation now has the name "eqn". First of all, we could simply solve the equation:

```
> dsolve(eqn,y(x));
```

$$y(x) = -\frac{1}{3}x^2 - \frac{2}{9}x - \frac{2}{27} + e^{(3x)} \_C1$$

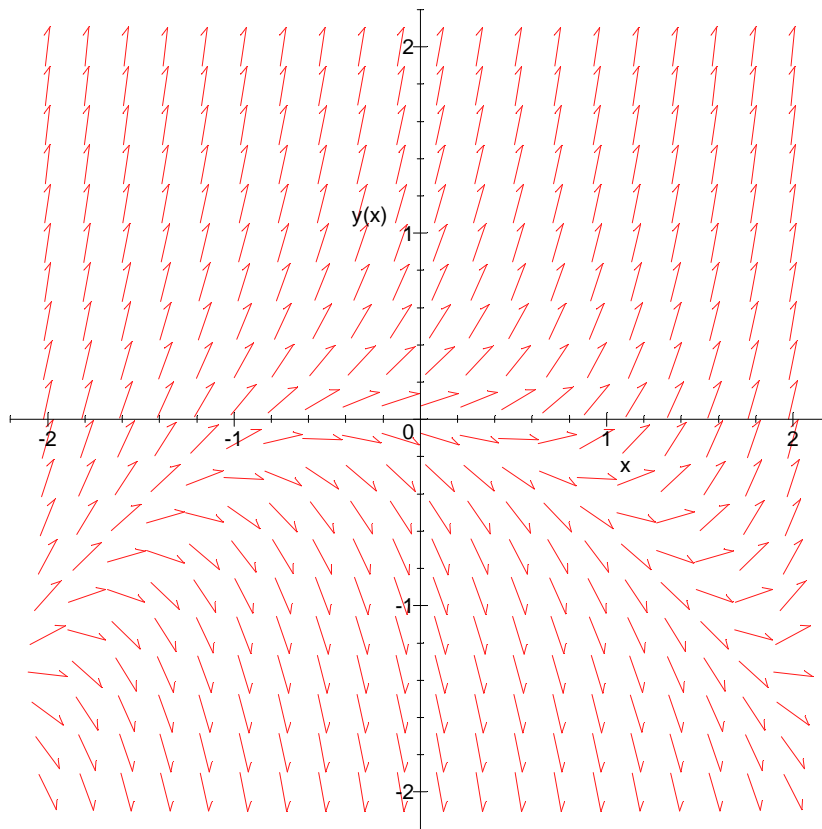
Notice that the dsolve command uses the standard ("what","how") syntax of Maple's main commands. The **\_C1** is the constant of integration (variables generated and named by the system always have names that begin with **\_**). We can substitute values for it, or else we could have begun by solving an initial-value problem:

```
> dsolve({eqn,y(0)=3},y(x));
```

$$y(x) = -\frac{1}{3}x^2 - \frac{2}{9}x - \frac{2}{27} + \frac{83}{27}e^{(3x)}$$

The geometry of first-order differential equations is important to understand ... this equation says that if the solution curve goes through the point **(x,y)** then its slope at that point must be **x^2+3y**. We can see this by drawing the "direction field" (as in section 15.1) using the Maple program "DEplot". For this equation:

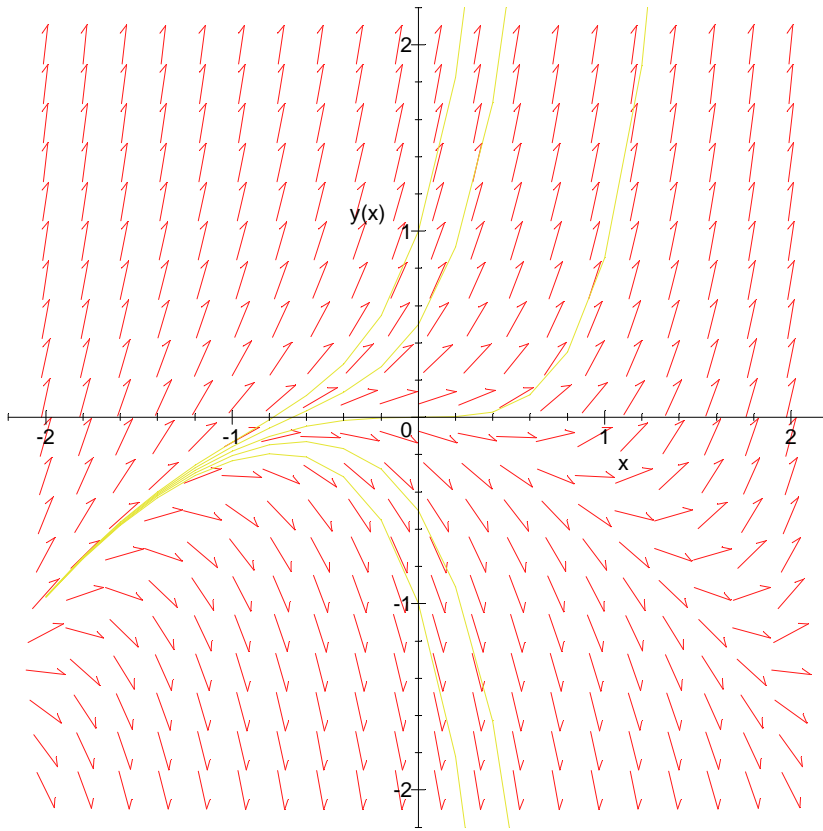
```
> DEplot(diff(y(x),x)=x^2+3*y(x),y(x),x=-2..2,y(x)=-2..2);
```



This syntax is tricky -- the program assumes that the equation is of the form  $y'=f(x,y)$  -- the function  $f(x,y)$  is given first, then the dependent variable (in this case  $y(x)$ ), then the ranges of the independent and dependent variables over which to plot.

It is often very enlightening to see the graphs of several solutions of the differential equation along with the direction field. To do this, we supply a set of initial points in between the two ranges:

```
> DEplot(diff(y(x),x)=x^2+3*y(x),y(x),x=-2..2,{[0,-1],[0,-0.5],[0,0]
,[0,0.5],[0,1]},y(x)=-2..2);
```



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