Solutions to Sample Math / Maple Project:

Exploring limits and plotting -- First steps in Maple

1. First we need to define h(x) and set the value of k:

```
> h:=x->sin(k*x)/x; k:=2;
```

$$h := x \to \frac{\sin(k\,x)}{x}$$

$$k := 2$$

Now we will use the "for" statement to make a table of values of h(x) with x approaching zero:

```
> for t from 1 to 6 do print(0.1^t,evalf(h(0.1^t))); od;
```

.1, 1.986693308

.01, 1.999866669

.001, 1.999998667

.0001, 1.999999987

.00001, 2.000000000

$.1\ 10^{-5}$, 2.000000000

It would seem from this that the limit of sin(2x)/x as x approaches zero is 2.

----

2. We will choose k=3, k=10, k=-5 and k=sqrt(2):

```
> k:=3: for t from 1 to 6 do print(0.1^t,evalf(h(0.1^t))); od;
```

.1, 2.955202067

.01, 2.999550020

.001, 2.999995500

.0001, 2.999999955

.00001, 3.000000000

$.1\ 10^{-5}$, 3.000000000

The limit of sin(3x)/x as x->0 seems to be 3.

```
> k:=10: for t from 1 to 6 do print(0.1^t,evalf(h(0.1^t))); od;
```

.1, 8.414709848

.01, 9.983341665

.001, 9.999833334

.0001, 9.999998333

.00001, 9.999999983

$.1\ 10^{-5}$, 10.00000000

The limit of sin(10x)/x as x->0 seems to be 10. There is a definite pattern here. I guess that the limit of sin(-5x)/x as x->0 will be -5.

```
> k:=-5: for t from 1 to 6 do print(0.1^t,evalf(h(0.1^t))); od;
```

.1, -4.794255386

.01, -4.997916927

.001, -4.999979167

.0001, -4.999999792

.00001, -4.999999998

$.1\ 10^{-5}$, -5.000000000

Good. Finally, we choose k=sqrt(2), and expect to get the sqrt(2) for the limit:

```
> k:=sqrt(2): for t from 1 to 6 do print(0.1^t,evalf(h(0.1^t))); od;
```

.1, 1.409504229

.01, 1.414166422

.001, 1.414213091

.0001, 1.414213557

.00001, 1.414213562

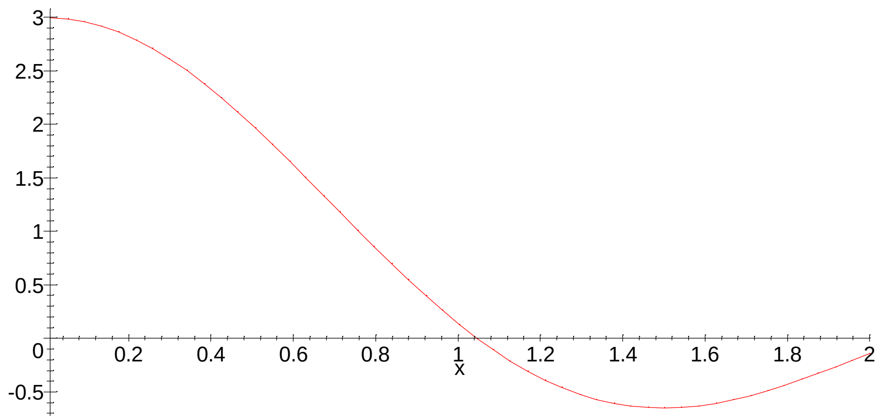
$.1\ 10^{-5}$, 1.414213562

The conclusion from all this is that the limit of sin(kx)/x as x->0 is k.

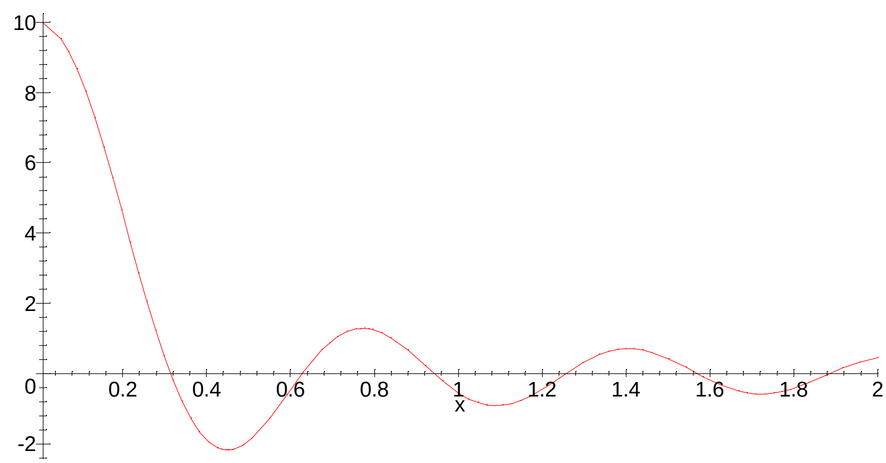
----

3. We go through the same k's and draw the plots. Since the k=2 plot was in the assignment sheet, we omit that one.

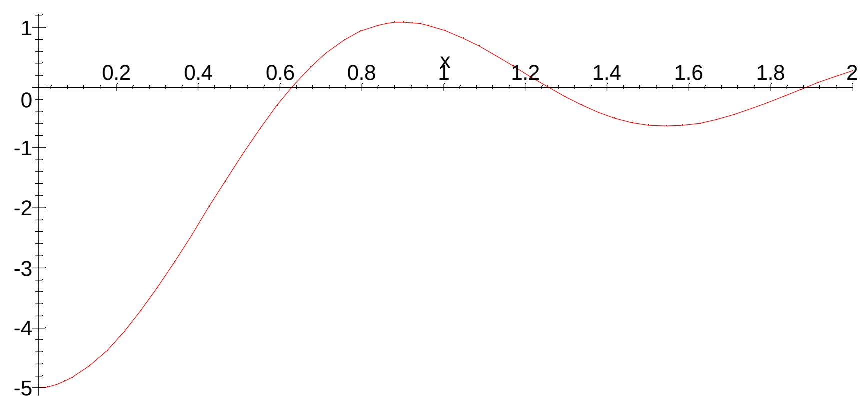
```
> k:=3: plot(h(x),x=0.01..2);
```



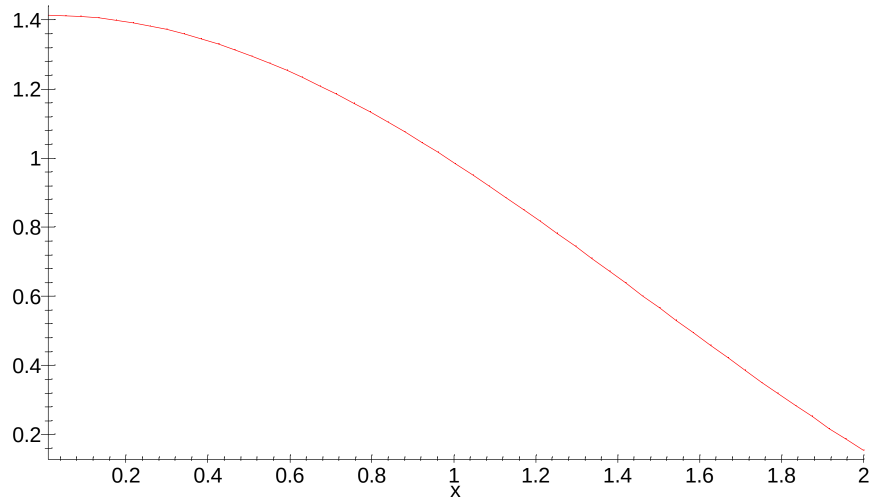

```
> k:=10: plot(h(x),x=0.01..2);
```




```
> k:=-5: plot(h(x),x=0.01..2);
```



```
> k:=sqrt(2): plot(h(x),x=0.01..2);
```



In each case, the conclusion of our numerical work in problem 2 was supported. Each time the limiting value of sin(kx)/x as x->0 appears to be k.

-----

4. We define the function s and proceed as we did in problems 2 and 3.

```
> s:=x->(1+k*x)^(1/x);
```

$$s := x \to (1 + k\,x)^{\left(\frac{1}{x}\right)}$$

```
> k:=1: for t from 1 to 6 do print(0.1^t,evalf(s(0.1^t))); od;
```

$$.1,\ 2.593742460$$
$$.01,\ 2.704813829$$
$$.001,\ 2.716923932$$
$$.0001,\ 2.718145927$$
$$.00001,\ 2.718268237$$
$$.1\ 10^{-5},\ 2.718280469$$

The limit as x -> 0, about 2.71828, looks a lot like the number e (denoted in Maple by exp(1)):

```
> evalf(exp(1));
```

$$2.718281828$$

```
> k:=2: for t from 1 to 6 do print(0.1^t,evalf(s(0.1^t))); od;
```

$.1, 6.191736422$

$.01, 7.244646118$

$.001, 7.374312390$

$.0001, 7.387578632$

$.00001, 7.388908321$

$.1\ 10^{-5}, 7.389041321$

This is a number I don't recognize. But since the limit when k was 1 was e, let's divide this number by e:

```
> evalf(7.389041321/exp(1));
```

2.718276392

So the limit (as x->0) appears to be e^2. Maybe the k ends up being the exponent this time. I predict that for k=3 I'll get e^3=exp(3).

```
> k:=3: for t from 1 to 6 do print(0.1^t,evalf(s(0.1^t))); od;
```

$.1, 13.78584918$

$.01, 19.21863198$

$.001, 19.99553462$

$.0001, 20.07650227$

$.00001, 20.08463311$

$.1\ 10^{-5}, 20.08544654$

```
> evalf(exp(3));
```

20.08553692

Looks good! Just to check, let's try k=-5:

```
> k:=-5: for t from 1 to 6 do print(0.1^t,evalf(s(0.1^t))); od;
```

$.1, .0009765625$

$.01, .005920529220$

$.001, .006653968579$

$.0001, .006729527022$

$.00001, .006737104780$

$.1\ 10^{-5}, .006737862775$

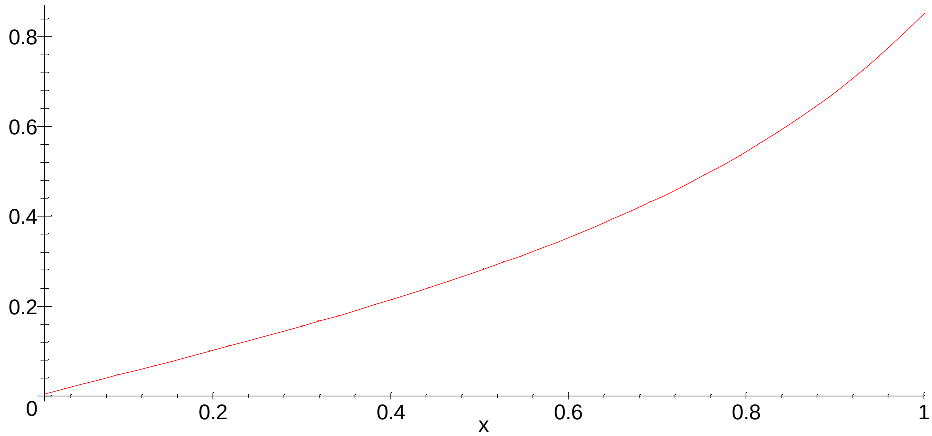
```
> evalf(exp(-5));
```

$.006737946999$

Conclusion: the limit of (1+k*x)^(1/x) as x->0 is E^k.

5. First we'll do the graph, then the numbers:

```
> f:=x->(sec(x)-1)/x: plot(f(x),x=0.01..1);
```



It looks as though the limit as x goes to zero is zero.

```
> for t from 1 to 6 do print(0.1^t,evalf(f(0.1^t))); od;
```

.1, .05020918000

.01, .005000200000

.001, .0005000000000

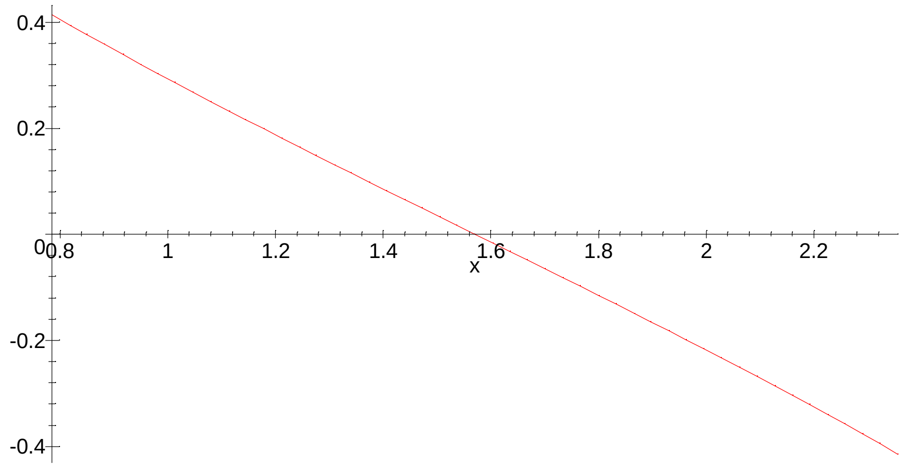
.0001, .00005000000000

.00001, 0

$.1\ 10^{-5}$, 0

I now believe that the limit is zero.

```
> g:=x->sec(x)-tan(x): plot(g(x),x=Pi/4..3*Pi/4);
```



Since Pi/2 is about 1.57, it looks like the limit of g as x -> Pi/2 is zero! Let's try the table:

```
> for t from 1 to 6 do
  print(evalf(Pi/2+0.1^t),evalf(g(Pi/2+0.1^t))); od;
```

1.670796327, -.050041707

1.580796327, -.00500006

1.571796327, -.0005003

1.570896327, -.000053

1.570806327, 0

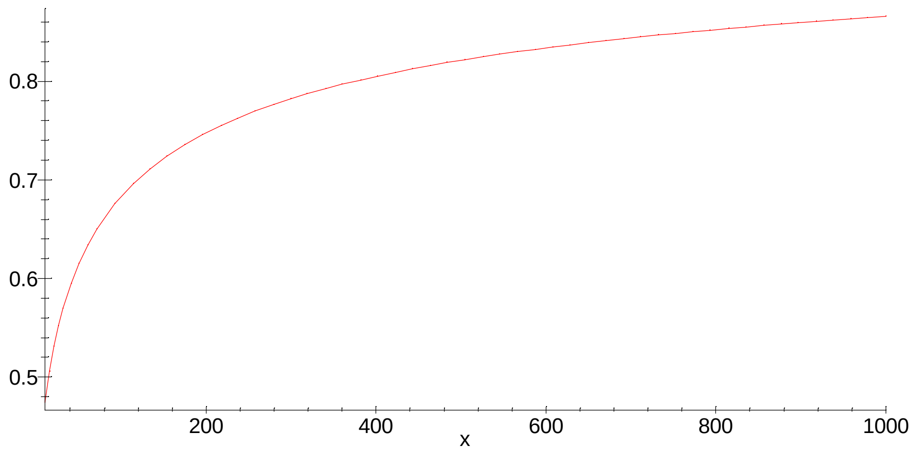
1.570797327, 0

Now I really believe that the limit of g as x approaches Pi/2 is zero.

```
> h:=x->sqrt(x+5)/(sqrt(x)+5):
```

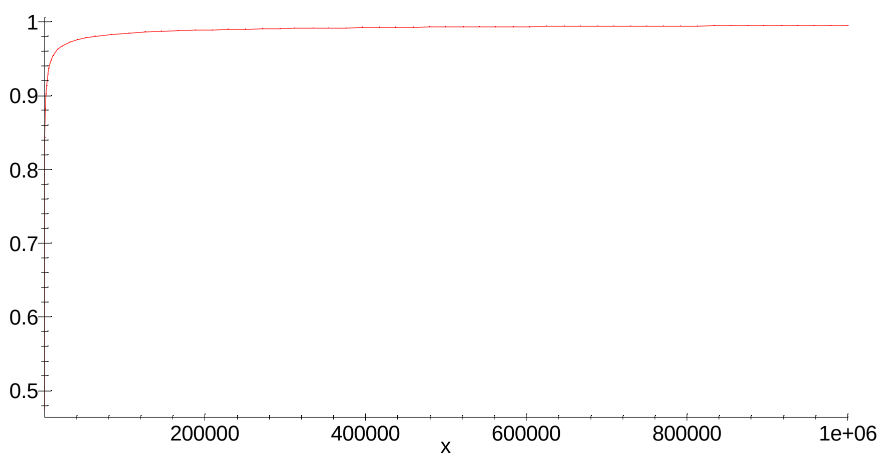
To get an idea of what happens as x->infinity, we'll try plotting over a large domain:

```
> plot(h(x),x=10..1000);
```



Even a larger range:

```
> plot(h(x),x=10..10^6);
```



Looks like the limit as x->infinity is 1. Let's do some numerics:

```
> for t from 1 to 10 do print(10^t,evalf(h(10^t))); od;
```

10, .4744978678

100, .6831300514

1000, .8656289313

10000, .9526190181

100000, .9844593310

1000000, .9950273632

10000000, .9984216070

100000000, .9995002749

1000000000, .9998419136

10000000000, .9999500035

```
> evalf(h(10^50));
```

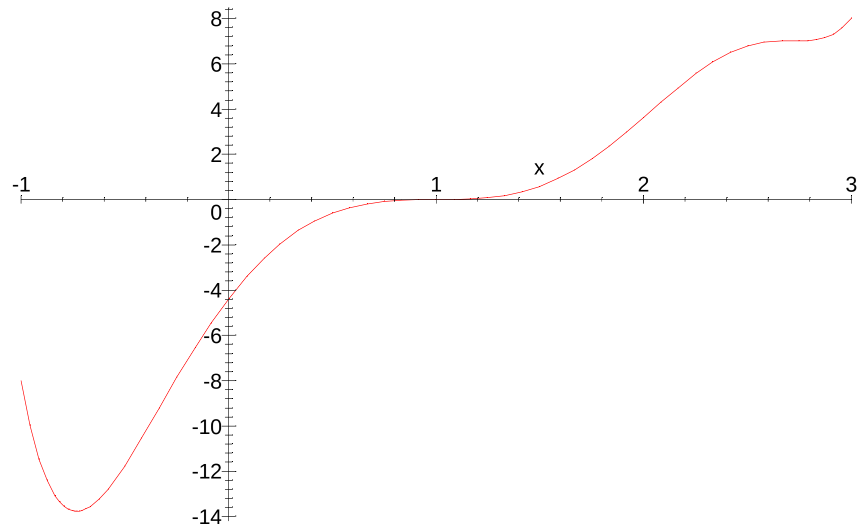
1.000000000

I accept 1 as the value of the limit.

----

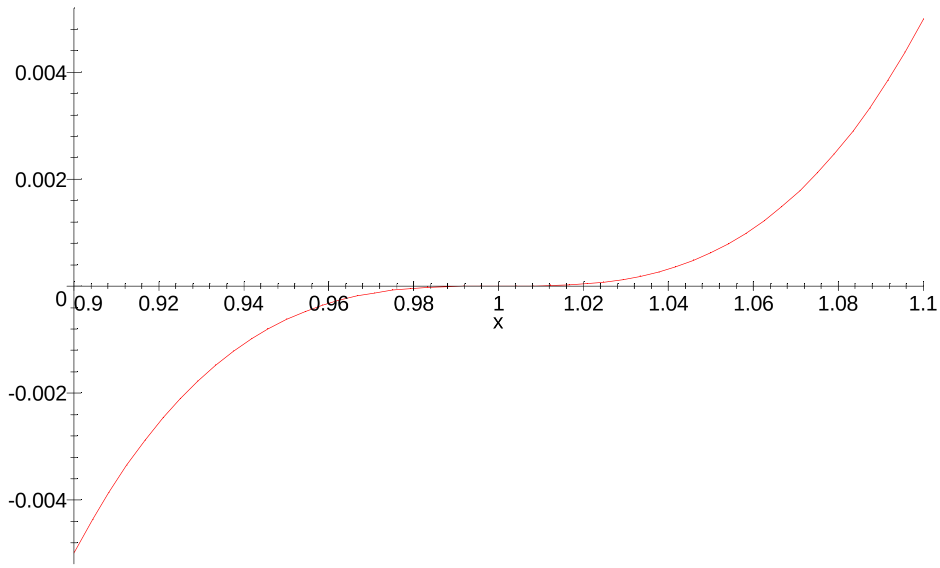
6. To do this one, we'll define the function g(x) and then plot over very small intervals containing the putative flex points:

```
> g:=x->(-35/8)+12*x - 9*x^2 - 2*x^3 + (25/4)*x^4 -
  5*x^5+3*x^6-x^7+x^8/8:
> plot(g(x),x=-1..3);
```



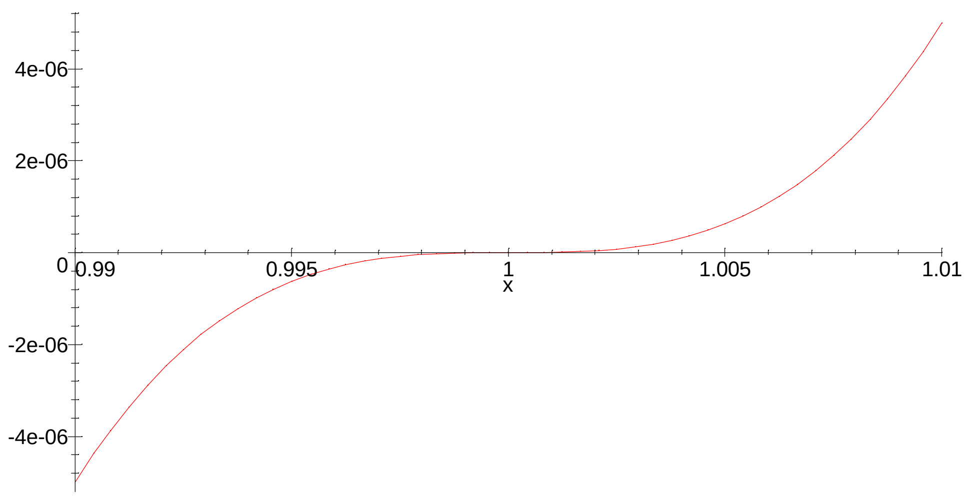
That was just to make sure we copied the function definition correctly. The first "flex" is at or near x=1:

```
> plot(g(x),x=0.9..1.1);
```



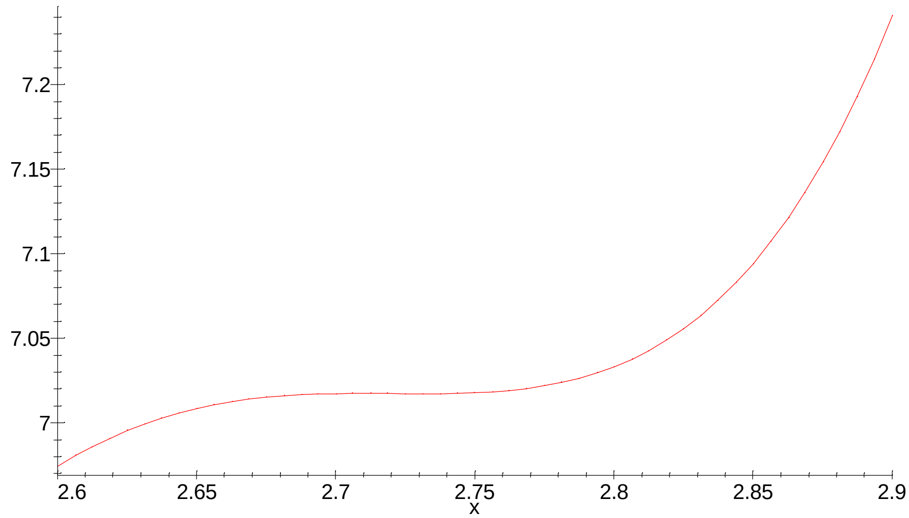
It still looks like a flex. Just to make sure, let's make the interval even smaller:

```
> plot(g(x),x=0.99..1.01);
```



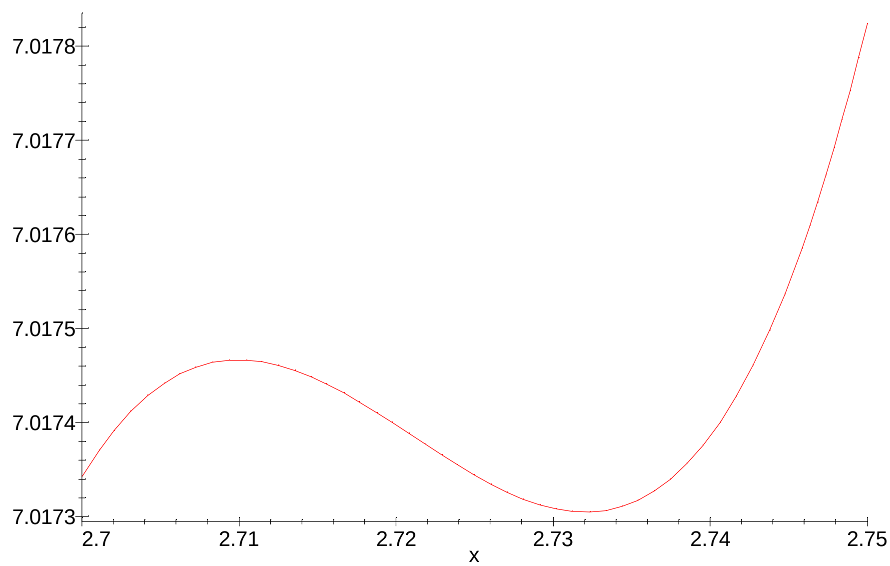
This seems like a genuine flex. Now for the other point, which seems to be between 2.5 and 3:

```
> plot(g(x),x=2.6..2.9);
```



Let's zoom in around 2.73:

```
> plot(g(x),x=2.7..2.75);
```



So this is not just a flex, the function has two turning points that we couldn't see before!