Math 312, Midterm 2

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1. **(5 points)** 51.
2. **(5 points)** 51 (the dual space has the same dimension as the original vector space; this was also on the first midterm).
3. **(10 points)** 1; a polynomial with zero derivative is constant.
4. **(10 points)** 50, by rank-nullity.
5. (Many answers suffice for these problems).
   (a) **(10 points)** No; $0 \notin A$.
   (b) **(10 points)** No; $0 \notin B$.
   (c) **(10 points)** Yes; $\langle \cdot, \cdot \rangle$ is positive-definite.
   (d) **(10 points)** Yes; D is the kernel of the linear map $f \mapsto \langle f, x^3 + x^2 + 1 \rangle$.
   (e) **(10 points)** No; $0 \notin E$.
   (f) **(10 points)** No; $0 \notin F$.
   (g) **(10 points)** No; $0 \notin G$.
6. (a) **(10 points)** 49; $H = \ker \mathrm{ev}_0 \cap \ker \mathrm{ev}_1$, where $\mathrm{ev}_x(f) := f(x)$. There is a polynomial f so that $\mathrm{ev}_0(f) \neq 0$. By rank-nullity, $\ker \mathrm{ev}_0$ has dimension 50. As $\mathrm{ev}_1 : \ker \mathrm{ev}_0 \to \mathbb{R}$ has nontrivial image (there is a polynomial f for which $\mathrm{ev}_0(f) = 0$ and $\mathrm{ev}_1(f) \neq 0$: for instance, x), rank nullity again shows that $\ker \mathrm{ev}_1$ restricted to $\ker \mathrm{ev}_0$ has dimension 49.
   (b) **(15 points)**

$$\int_0^1 Df(x)g(x)\,dx = f(x)g(x)\Big|_0^1 - \int_0^1 f(x)Dg(x)\,dx.$$

But

$$f(x)g(x)\Big|_0^1 = 0$$

so

$$\langle Df(x), g(x)\rangle = \langle f(x), -Dg(x)\rangle.$$