

Math 312, Midterm 1

Aaron M. Silberstein

February 12, 2013

You have 50 minutes to complete this midterm.

1. Let $A : \mathbb{R}^{30} \rightarrow \mathbb{R}^{40}$, $B : \mathbb{R}^{40} \rightarrow \mathbb{R}^7$ and $C : \mathbb{R}^7 \rightarrow \mathbb{R}^{57}$ be linear transformations.

(a) **(10 points)**. What are the possible dimensions of the image of $C \circ B \circ A$?

(b) **(10 points)**. What are the possible dimensions of the kernel of $C \circ B \circ A$?

(c) **(10 points)**. Can $C \circ B \circ A$ be injective? Can $C \circ B \circ A$ be surjective? Can $C \circ B \circ A$ be invertible? Use the answers to (a) and (b) to justify your answers.

2. **(10 points)**. Write down a basis of \mathcal{P}_4 , the vector space of polynomials with real coefficients of degree ≤ 4 .

3. **(10 points)**. What is the dimension of the dual space of \mathcal{P}_4 ?

4. Consider the linear map

$$\text{ev}_3 : \mathcal{P}_4 \rightarrow \mathbb{R}$$

given by

$$\text{ev}_3(f) = f(3).$$

(a) **(5 points)**. What is the dimension of the kernel of ev_3 ?

(b) **(5 points)**. What is the dimension of the image of ev_3 ?

(c) **(10 points)**. Give a basis of the kernel of ev_3 (Hint: any polynomial of the form $(t - 3)g(t)$ is in the kernel).