

Math 312, Midterm 1 Solutions

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- (a) **(10 points)**. $0 \leq \dim \operatorname{im} C \circ B \circ A \leq 7$.

(b) **(10 points)**. By rank-nullity, $23 \leq \dim \ker C \circ B \circ A \leq 30$.

(c) **(10 points)**. $C \circ B \circ A$ cannot be injective, as $\dim \ker$ cannot be zero. $C \circ B \circ A$ cannot be surjective, as $\dim \operatorname{im}$ cannot be 57. As $C \circ B \circ A$ cannot be injective or surjective, it cannot be invertible.
- (10 points)**. Many would work; for instance, $\{1, t, t^2, t^3, t^4\}$.
- (10 points)**. $\dim \mathcal{P}_4 = 5$ so $\dim \mathcal{P}_4^* = 5$ as well.
- (a) **(5 points)**. 4.

(b) **(5 points)**. 1.

(c) **(10 points)**. For example: $\{(t-3), t(t-3), t^2(t-3), t^3(t-3)\}$.