FEATURE

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Endpaper: How to Compute Determinants

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During one of my years in graduate school in Israel, I was a teaching fellow for a class on linear algebra. I found the job annoying for two reasons: On one hand, the students were primarily non-math majors. But more importantly, my class started at eight in the morning, which did not rhyme well with my lifestyle at the time. As a result, I could not bring myself to prepare my section in advance. Instead I improvised each time....

One day I found myself explaining determinants. "You know, for a generic matrix a determinant is never zero. Somebody, give me an example of a matrix!" The class produced no reply. They were no less sleepy than I was. In fact, not only were they asleep but they were suspicious as well. They did not want to risk giving a matrix which by misfortune would have a zero determinant, with the gloomy title of "degenerate" attached to it.

So I proceeded: "OK, let's take the first matrix that comes to mind."

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

I set about computing the determinant by the usual formula. I was never good with computations and, once again, I was especially sleepy:

$$1 \cdot 5 \cdot 9 - 2 \cdot 4 \cdot 9 \pm 3 \cdot 4 \cdot 8 + \dots .$$

It took me a good 10 minutes. And what a shock, the determinant was zero! "I must have made a mistake," I told the class. I ran through the calculations once more, checking every step. Another 10 minutes passed. Zero again!

I tried to save myself. "OK, but sometimes the determinant is zero. Sorry. But now let's take a *really* generic matrix."

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

Another lengthy computation. . . .

At the end of that semester I was forced to enroll in a special seminar for delinquent instructors.

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