Math 371, Spring 2013, PSet 1

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1. Give two nonisomorphic varieties of finite type over a given field with isomorphic, nontrivial étale fundamental groups.

2. Give two nonisomorphic fields of the same characteristic with isomorphic, nontrivial absolute Galois groups.

3. Give two fields of different characteristic with isomorphic, nontrivial absolute Galois groups.

4. Let L_1 and L_2 be two number fields, which are normal extensions over $\mathbb{Q}$. Let S_1 and S_2 be the set of primes of $\mathbb{Q}$ which split completely in L_1 and L_2, resp. If $S_1 = S_2$ then L_1 is isomorphic to L_2 (hint: use the Čebotarev density theorem).

5. Prove that $\hat{\mathbb{Z}} \simeq \prod_{p \text{ prime}} \mathbb{Z}_p$ and $\hat{\mathbb{Z}}^\times \simeq \prod_{p \text{ prime}} \mathbb{Z}/(p-1)\mathbb{Z} \times \mathbb{Z}_p$.

6. Prove that the only valuations on $\mathbb{Q}$ are the p-adic valuations.

7. What are all the valuations on $\mathbb{Q}(t)$?

8. Classify the fields with no nontrivial valuation.

9. Classify the valuations on any subfield of $\overline{\mathbb{Q}}$.

10. Prove that the Brauer group of a finite field is trivial.

11. Prove that every self-homeomorphism of the underlying Zariski topological space of $\mathbb{P}^2$ over $\mathbb{C}$ is induced by an automorphism (not necessarily over $\mathbb{C}$) of schemes. Is this true for any other algebraic surfaces?