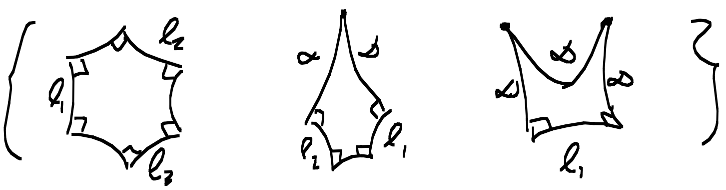
Block TQFTs 10/31/8

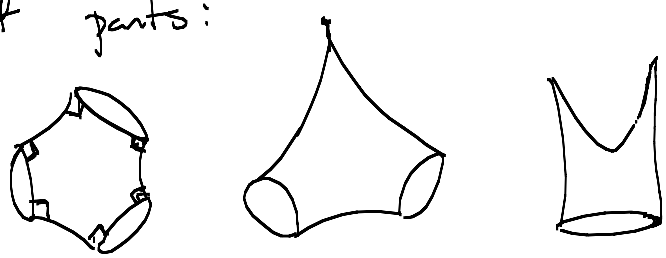
T_g^s = Teichmuller space : set of Riemann surfaces of genus-g and s-marked pts.

such surfaces are built out of doubled right-angled hexagons.



Double them and glue along unparametrized edges to get pants:

$\chi(P_i) = -1$



P_0 P_1 P_2

$$\chi(F_g^s) = 2-2g-s$$

If we have F_g^s s.t. $F_g^s = aP_0 + bP_1 + cP_2$

$$\chi(F_g^s) = \chi(aP_0 + bP_1 + cP_2) = -(a+b+c)$$

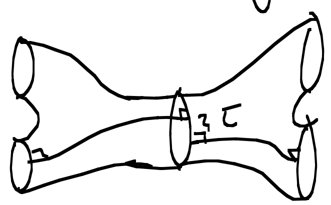
$$\# \text{ of parameters} = \frac{1}{2}\Big(3(a+b+c) - \underbrace{(b+2c)}_{\#\text{ of marked pts} = s}\Big)$$

$$= \frac{1}{2}\Big(3\underbrace{(a+b+c)}_{s+2g-2} - s\Big)$$

$$= 3g-3+s$$

The constituents contribute $3g-3+s$ parameters

Then we can glue w.r.t. to any rotation of boundary circles!



After fixing geodesics $\perp$ to circle τ is a parameter which measures displacement.

<u>Theorem</u> (Nielsen, Fenchel)

$$T_g^s \cong (\mathbb{R}^{>0} \times \mathbb{R})^{3g-3+s}$$

Mapping class group: Γ_g^s = Isotopy classes of diffeos fixing marked pts.

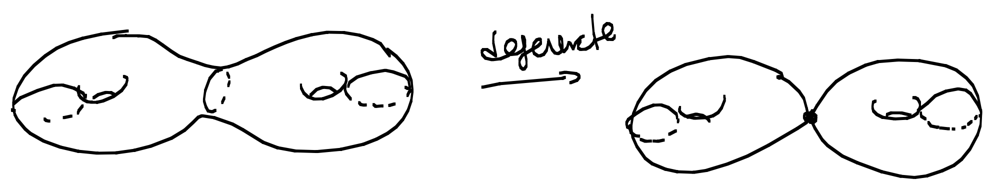
acts on T_g^s proper and discontinuities $\Rightarrow$ isotropy is finite

$$\mathcal{M}_g^s \equiv T_g^s / \Gamma_g^s = \text{orbifold } g \geq 2.$$

<u>Compactification</u>

<u>Deligne-Mumford</u> <u>compactification</u> (+Thurston geometrically)

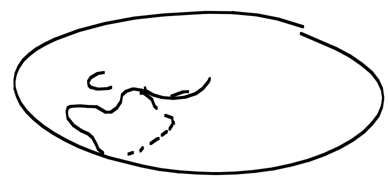
Let some of the ℓ_i's go to 0.

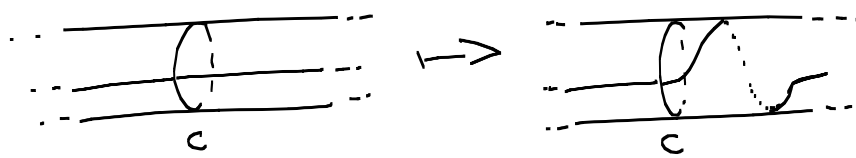


By throwing in nodal curves we get a cpct. spc after the quotient: $\overline{M}_g^s = \overline{T}_g^s / \Gamma_g^s$

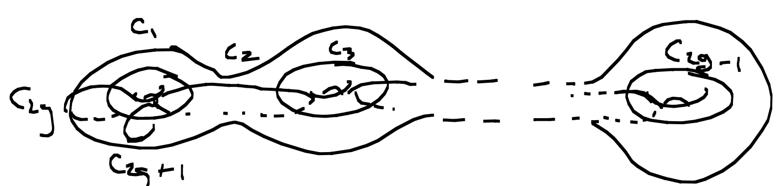
<u>generators of</u> Γ_g^s: If f surface, C simple closed curve: define Dehn twist about C:

cut out C, rotate by 2π and glue back together



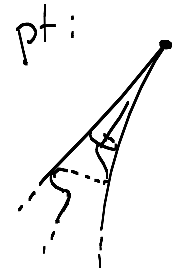


Γ_g^s is finitely presented by a finite # of Dehn twists



gen'td by $2g+1$ Dehn Twists.

(for marked pts. probably add an extra twist around each pt:

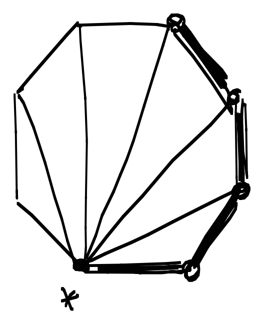


Now let F be a closed surface, genus s marked pts.
Let $s=1$ for simplicity and $*$ be the marked pt.
Consider the isotopy class of a family $\alpha_0, \dots, \alpha_k$ of simple closed curves, rel $*$, and suppose they satisfy

1) Two of these meet only at $*$
2) No two are homotopic rel. $*$
3) None are null homotopic.

Such a family is called a rk k arc system.

<u>Note</u>: maximal rk is $6g-4$. :



$$\Big\} \ 2g + (4g-3) = \boxed{6g-3}$$

but

$$2(3g-3+1)$$

Form a simplicial complex of dim $6g-4$ by defining a k-simplex to be an arc. cmplx of rk k $\langle \alpha_0, \dots, \alpha_k \rangle$
$\langle \beta_0, \dots, \beta_e \rangle$ is a face of $\langle \alpha_0, \dots, \alpha_k \rangle \Leftrightarrow \{\beta_i\} \subseteq \{\alpha_i\}$
This forms a simplicial cmplx. A point in the geometric realization is given by an arc cmplx $\langle \alpha_0, \dots, \alpha_k \rangle$ together w/ weights $w_0, \dots, w_k \geq 0$ $\quad \sum w_i = 1$.

$\Gamma_g^{1}(s)$ acts on this simplicial cmplx $\varphi : F \to F$

$$\varphi(\langle \alpha_0, \ldots, \alpha_k \rangle) = (\langle \varphi(\alpha_0), \ldots, \varphi(\alpha_k) \rangle$$

allows to calc. homology of Teichmuller spc.