Block TFTs 11/3/8

Recall: finding a triangulation of Teichmüller space.

Define a <u>rank</u> <u>k</u> <u>arc system</u> $\Gamma^1_{g,*}$

$\langle \alpha_0, \ldots, \alpha_k \rangle$ α_i isotopy class rel. $*$ of simple closed curves, only intersecting at $*$ and satisfying

1) $[\alpha_i] \neq [\alpha_j]$ $i \neq j$
2) $[\alpha_i] = e$

The maximal k is $6g-4$. Let A be the abstract simplicial complex defined by simplices $\langle \alpha_0, —, \alpha_k \rangle$ and $\langle \beta_0, —, \beta_e \rangle$ is a face of $\langle \alpha_0, —, \alpha_k \rangle$ iff

$\{[\beta_i]\} \subseteq \{[\alpha_j]\}$

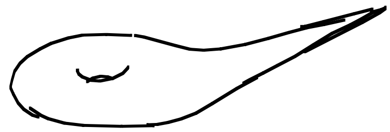
$|A|$ is parametrized by an <u>arc system</u> $\langle \alpha_0 - \alpha_k \rangle$ together w/ weights $w_0, —, w_k \geq 0$ $\sum w_i = 1$

A family of closed curves is said to <u>fill</u> F if each component of the complement is simply connected.

Let $A_\infty = \{\langle \alpha_1 - \alpha_k \rangle\}$ s.t. $\{\alpha_0, —, \alpha_k\}$ don't fill F.

Γ_g^1 acts on A by $\varphi(\langle \alpha_0, —, \alpha_k \rangle) = \langle \varphi(\alpha_0), --, \varphi(\alpha_k) \rangle$

Theorem There is a homeomorphism $\omega : T_g' \longrightarrow |A| - |A_\infty|$ which preserves the action of the mapping class group.

example) T_1'  $= F_1'$ $(6g-4=2)$

Clearly, a single curve can't fill F and an arc system of rk 1 or 2 must fill F

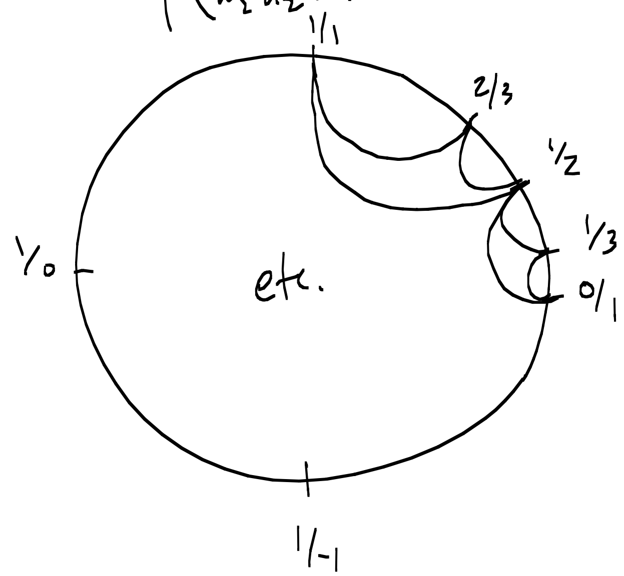
so, A_∞ = vertices of A = null homotopic closed curves in A.

$\pi_1(F, *) = \mathbb{Z}^2$. Let α, β be a basis. Any simple closed curve looks like $\gamma = m\alpha + n\beta$ $(m,n) = 1$

We assign to $\gamma \rightsquigarrow m/n \in \mathbb{Q} \cup \{\infty\}$.

$A_\infty = \mathbb{Q} \cup \{\infty\}$. (m_1, n_1) defines the same elt. as (m_2, n_2)

$\Longleftrightarrow \left| \begin{pmatrix} m_1 & n_1 \\ m_2 & n_2 \end{pmatrix} \right| = \pm 1$ so, $A = \mathbb{H}^2 \cup \mathbb{Q} \cup \{\infty\}$

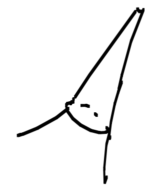


We now define ω: Let X be a complete hyperbolic manifold of finite area and one cusp. Let f be a homeo.

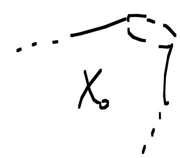
$$f: X \to F \setminus \{*\} \quad \text{so, } (X, [f]) \in T_g^1$$

We want to associate a pt. $\omega(X,[f]) \in |A| \setminus |A_\infty|$

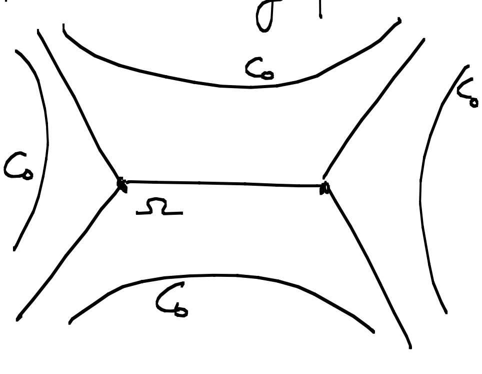
Let $C_0 \subseteq X$ be a <u>horocycle</u> about the cusp



Let $X_0 = \{$open punctured disk outside the horocycle$\}$



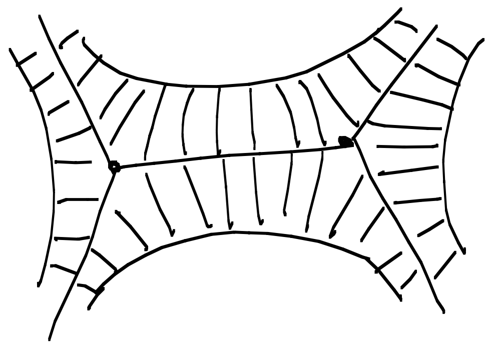
Let $\rho : X_0 \to \mathbb{R}^{\geq 0}$ be the distance to the horocycle. Let $C_t = \rho^{-1}(t)$ For small t, C_t is smooth but in general there are singularities coming from pts on C_t which have 1 minimizing pt. in C_0.



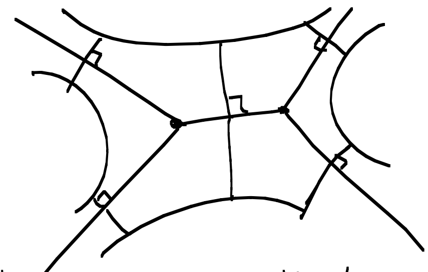
These singularities form a subset Ω which is a connected graph

$X_0 \setminus \Omega$ = punctured disk.

Consider geodesics $\perp$ to C_0:
These will form a singular "foliation": F of X_0
Each non-singular leaf will be the union of two geodesic segments joining C_0 to Ω:



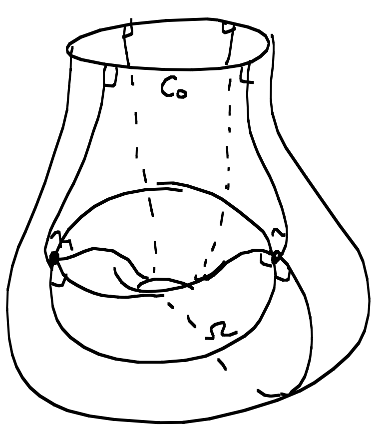
For each edge of Ω, $\exists$ unique geodesic $\bar{\alpha}_i$ $\perp$ to Ω:



α_i goes through the cusp, and the collection of these α_i's form an arc system.

Now we need to assoc. the weights: Look at the leaves of the foliation that meet the singular points of Ω. The intersection of these leaves w/ C_0 divides C_0 into segments. Each of these segments meet exactly one $\bar{\alpha}_i$. And the two segments that α_i meets have the same length l_i. Set $w_i = l_i/l$ where $l = \sum l_i$. Then the map ω;

$$\omega(X,[f]) = (\langle \alpha_0, -, \alpha_k \rangle, w_0 - w_k)$$

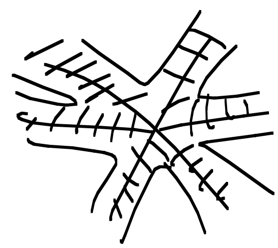


__Define__ Let S be a set. A cyclic ordering of S is a $\sigma : S \to S$, s.t. $\forall x \in S$ $\sigma^k(x), k = 0, \dots, n = S$

__Define__ A __ribbon graph__ is a graph s.t. edge set of each vertex is cyclically ordered.

Any planar graph is a ribbon graph. An embedding of a graph Γ into oriented surface gives Γ the structure of a ribbon graph.

__Lemma__ Every ribbon graph can be embedded into an oriented, closed surface s.t. the orientation induces the original cycle order on the edge sets.



From these constituents you glue together to form a surface w/ boundary. And glue in discs into boundary components.