Challenge Problems: (related to course material to some degree: not for extra credit but for interest/fun; the more asterisks, the more difficult the problem in my estimation, although this measure is really rough)

1**) Can the arc of a parabola inside a circle of radius 1 have length greater than 4? What about a square of length 2?

2**) Show that if given a set of unit vectors in the place, there is a way to sum them with coefficients +/- 1 so that the resultant vector is of length less than or equal to $\sqrt{2}$.

3) Given 5 points on a sphere, show that there is a closed hemisphere which contains 4 of them.

4**) Given 3 distinct points on the sphere compute the surface area of the spherical triangle defined by them. (This is the triangle on the sphere whose sides are the great circles definied by pairs of points). If it is easier compute this area in terms of the three angles of the spherical triangle.

5*) Given an axis in $\mathbb{R}^3$ described by spherical angles (θ, ϕ), and an angle η, produce a 3×3 matrix which affects a rotation of $\mathbb{R}^3$ by angle η around the given axis.

6) Find the volume of the region in $\mathbb{R}^3$ with $(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2)$

7) Prove that a 3×3 orthogonal matrix A will map the sphere into itself, i.e. if x is a point on the sphere, Ax is also. Then show that any such map will have a fixed point, i.e. there is some point x on the sphere such that Ax=x.

8*) Compute the surface area and volume of a ellipsoid of axial lengths a,b,c.

9) Write out a formula for spherical coordinates in $\mathbb{R}^4$. What about $\mathbb{R}^n$ for some arbitrary n?

10*) compute the volume formula for an n-sphere: We already know $2\pi r$ for $n = 1$, $4\pi r^2$ for $n = 2$. What about n=3?, What about arbitrary n?