

Return to example.

ex QC_X ← generic coherent sheaves on X
↑ some space

start w/ multls of dim $n-1$.

$$F: M^{n-1} \rightarrow QC_{X^M} \cong M \otimes QC_X$$

$$M \downarrow \rightsquigarrow X^* \cong X$$

using this eval. map
get adjunction:

$$QC_{X^M} \xrightleftharpoons[\text{ev}_*]{\text{ev}^*} QC_X$$

$$X \cong X^* \rightarrow X^M \downarrow \text{ev} X$$

$$X = \text{spec } A$$

so, compute $\text{ev}_* \text{ev}^*(1 = \mathcal{O}_X)$

$$\begin{matrix} (M \otimes A)\text{-mod} \\ \uparrow \text{Ind}(\text{ev}) \downarrow \text{Res} (* \otimes A \rightarrow M \otimes A) \\ A\text{-mod} \end{matrix}$$

$= \text{ev}_* \mathcal{O}_{X^M}$ is commutative DGA in QC_X

Thus identifies $QC_{X^M} \cong \text{Mod}_{\text{ev}_* \mathcal{O}_{X^M}}(QC_X)$

↑ QC sheaves on X w/ an action of $\text{ev}_* \mathcal{O}_X$

$$M = S^1, X = \text{spec } A$$

$$\begin{matrix} S^1 \otimes A = \text{HH}_X A \\ \parallel \\ \mathcal{O}(X^{S^1}) \end{matrix}$$

$$\begin{matrix} S^0 \rightarrow * \\ \downarrow \quad \downarrow \\ * \rightarrow S^1 \end{matrix} \quad \begin{matrix} \text{apply } (\cdot) \\ \rightsquigarrow \end{matrix}$$

$$\begin{matrix} X^{S^1} \xrightarrow{\text{ev}} X \\ \downarrow \text{hoPB} \quad \downarrow \\ X \rightarrow X^{S^0} \end{matrix} \quad \begin{matrix} \text{spec} \\ \hookrightarrow \end{matrix} \quad \begin{matrix} S^0 \otimes A \cong A \otimes A \rightarrow A \\ \downarrow \quad \downarrow \\ A \rightarrow S^1 \otimes A \end{matrix}$$

use push-pull formula:

$$\begin{matrix} X'' \xrightarrow{f'} X' \\ \downarrow \text{PB} \quad \downarrow j \\ X'' \xrightarrow{f} Y \end{matrix} \quad \longrightarrow \quad \begin{matrix} g^* f' = f_* g^* \end{matrix}$$

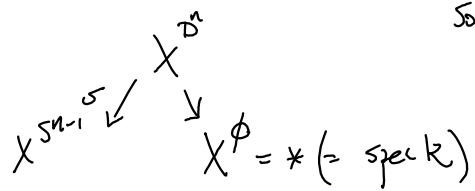
$$\Rightarrow \underline{\text{ev}_* \text{ev}^* = \Delta^* \Delta_*}$$

$$\Delta^* \Delta_* \mathcal{O}_X = \text{HH}_*(\mathcal{O}_X) \leftarrow \text{HH-sheaf.}$$

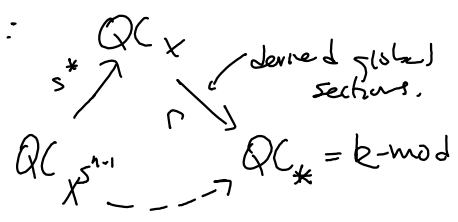
$$\Rightarrow QC_{X^{S^1}} = \text{Mod}_{\Delta^* \Delta_* \mathcal{O}_X}(QC_X) \cong \text{Mod}_{\text{HH}_*(X)}(QC_X).$$

Cases: $0 \xrightarrow{D^n} \emptyset^{n-1}$
 $S^n \xrightarrow{D} \emptyset$ empty $n-1$ multls.

$$S^{n-1} \hookrightarrow D^n \hookrightarrow \emptyset^{n-1}$$



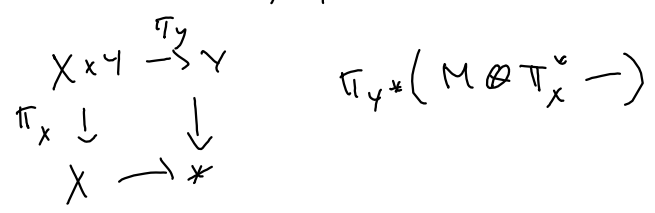
So, D^n yields:



Note: "Thm" X, Y nice algebraic spaces

$F(D^n_{S^{n-1} \rightarrow \emptyset}) \cong \Gamma_0 S^+$
a functor

\exists product functor: $QC_{X \times Y} \xrightarrow{\sim} \text{Fun}(QC_X, QC_Y)$ [see Toën]



So, via the above thm, $\Gamma_0 S^+$ corresponds to a sheaf on $X^{S^{n-1}}$.

$QC_{X^{S^{n-1}}} \xrightarrow{\cong} \text{Fun}(QC_{X^{S^{n-1}}}, k\text{-mod})$
 \downarrow
 $M \rightarrow \Gamma(M \otimes -)$ find M s.t.
 $\Gamma(M \otimes -) \cong \Gamma_0 S^+(-)$

$\Gamma_{X^{S^{n-1}}}(M \otimes -) = \Gamma_X S_+ (M \otimes -)$

answer:
 $M = S_+ \mathcal{O}_X$

$\Gamma_{X^{S^{n-1}}}(S_+ \mathcal{O}_X \otimes (-)) = \Gamma_{S_+}(\mathcal{O}_X \otimes S^+(-)) = \Gamma_X S_+ S^+(-)$
projection formula

 $= \Gamma_{X^{S^{n-1}}} S^+(-)$

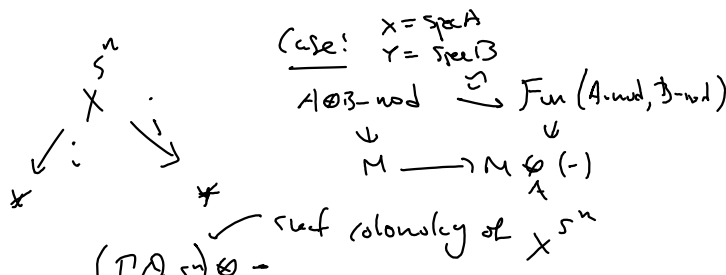
under this correspondence,

$\Gamma_0 S^+ \cong S_+ \mathcal{O}_X$

$\begin{pmatrix} X \leftarrow M \\ \downarrow f \\ Y \leftarrow N \end{pmatrix} \quad f_* M \otimes N \cong f_*(M \otimes f^* N)$

Philosophy: what happens on sites should determine the theory.

$F(S^n_{\phi^{n-1} \rightarrow \phi^{n-1}}) \in \text{Fun}(k\text{-mod}, k\text{-mod})$



in this case $j_* i^+(-) = j_* i^-(k) \otimes - = (\Gamma \mathcal{O}_{X^{S^n}}) \otimes -$

try to push further : $F(M) \equiv \Gamma(\mathcal{O}_{X^m})$

