MATH 114 Exam 2

Wednesday, 16 November 2011

Name:

This exam consists of 5 multiple-choice questions and 4 true-false questions. Show *all* your work. You will receive credit for a correct answer only if *your work is shown* and *your work supports your answer*. No credit will be given for correct answers which are not supported by work.

When you have finished, please copy your answers to this page.

You will have 50 minutes to complete this exam.

Problem	1	2	3	4	5	6.i	ii	iii	iv
Answer									

1. Let $f(x, y) = x \cos y$. Where does $f(x, y)$ achieve local maxima?

A. $(0, \pm k\pi)$ for $k = 0, 1, 2, \ldots$ B. $(0, \frac{\pi}{2} \pm k\pi)$ for $k = 0, 1, 2, \ldots$ C. $(1, 0)$

D. $(1, \pi)$ E. There are no local maxima. F. $(0, 0)$

2. What is the linearisation for $g(x, y) = e^{x+y^2}$ at $(0, \sqrt{\ln 5})$?

A. $L(x, y) = 5 + 5x + 10\sqrt{\ln 5}(y - \sqrt{\ln 5})$

B. $L(x, y) = 5 + e^{x+y^2}x + 2ye^{x+y^2}(y - \sqrt{\ln 5})$ C. $L(x, y) = 5 + 10\sqrt{\ln 5}$

D. $L(x, y) = (\ln 5)x + \sqrt{\ln 5}(y - 5)$ E. $L(x, y) = 5 + (1 + 2\sqrt{\ln 5})e^{x+y^2}(x + y)$

3. Find the volume of the region bounded by the xy plane and the elliptic paraboloid $z = 4 - x^2 - y^2$.

A. $4\pi^2$ B. 1 C. $\frac{32}{3}\pi$ D. 8 E. $\frac{\pi}{4}$ F. 8π

4. Find a point that is closest to the origin on the curve $y = \frac{2}{x}$. You must use a technique discussed in class.

A. $(0, 0)$ B. $(\sqrt{2}, \sqrt{2})$ C. $(\sqrt{2}, \frac{\sqrt{2}}{2})$ D. $(6, \frac{1}{3})$ E. $(2, 1)$ F. $(1, 2)$

5. The cylinders $x^2 + y^2 = 9$ and $y^2 + z^2 = 25$ intersect in a curve. The point $(0, 3, 4)$ lies on that curve. Give a vector tangent to the curve of intersection at $(0, 3, 4)$.

A. $\mathbf{k}$ B. $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ C. $\mathbf{i} + \mathbf{k}$ D. $\mathbf{j} + \mathbf{k}$ E. $\mathbf{i}$ F. $2\mathbf{i} + \mathbf{k}$

6. TRUE or FALSE. For each of the following statements, indicate whether it is true (T) or false (F). Support your answers.

   i The function $f(x, y) = x \cos y$ in problem 1 has no local minima.

   ii The function $g(x, y) = \begin{cases} \frac{xy}{|xy|} & \text{if } xy \neq 0 \\ 0 & \text{if } xy = 0 \end{cases}$ is continuous.

iii $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}}\int_{x^2}^{\sqrt{6-x^2}}(\sin x)dydx = \int_0^2\int_{-\sqrt{y}}^{\sqrt{y}}(\sin x)dxdy + \int_2^{\sqrt{6}}\int_{-\sqrt{6-y^2}}^{\sqrt{6-y^2}}(\sin x)dxdy$

(*Hint.* DO NOT attempt to evaluate any of the integrals.)

iv Let $h(x, y, z) = \frac{y}{x} + z^2$. A particle leaving the point $(1, 1, 2)$ and heading in a straight line towards $(3, 3, 2)$ is moving tangentially to the level surface $h(x, y, z) = 5$ when it leaves the point $(1, 1, 2)$.