MTH 114 Exam 2 Practice

1. Consider the iterated integral $\int_1^e \int_0^{\ln x} y dy dx$.
   (a) Sketch the region of integration.

   (b) Rewrite the integral, reversing the order of integration.

   (c) Evaluate the integral.

2. Suppose the temperature of a metal plate is given by $T(x, y) = x\sin(2y)$, where x and y are cartesian coordinates for the plate.

   (a) Suppose the probe is moving along the circle of radius 1, centred at the origin, according to $x = \cos 2t, y = \sin 2t$. How fast is the temperature probe's reading changing when it reaches the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$?

   A. $\sqrt{2}$ B. $\sqrt{3}$ C. $\sqrt{2}\cos\sqrt{3}$

   D. $-\sqrt{3}\sin\sqrt{3} + \cos\sqrt{3}$ E. $\frac{\pi}{6}$ F. $\frac{1}{2}\sin\sqrt{3}$

   (b) Starting from $(-1, -1)$, in what direction should the temperature probe be moved to give the greatest increase in temperature?

   A. $\langle\frac{3}{5}, \frac{4}{5}\rangle$ B. $\langle-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\rangle$ C. $\langle\frac{\sin(-2)}{\sqrt{1+3\cos^2(-2)}}, \frac{-2\cos(-2)}{\sqrt{1+3\cos^2(-2)}}\rangle$

   D. $\langle 1, 0\rangle$ E. $\langle 0, -1\rangle$ F. $\langle\frac{1}{2}, \frac{\sqrt{3}}{2}\rangle$

3. Let $f(x, y) = \sin(x^2 - y^2)$. Which of the following planes is parallel to the tangent plane to the graph $z = f(x, y)$ when $x = 1$ and $y = 1$?

A. $3x + 4(y - 1) + z = 0$ B. $-2(x + 7) + 2y + z = 5$ C. $z = 1$

D. $x + y + z = 0$ E. $-3x + 4y - z = 0$ F. $2x - 2y = 9$

4. Find the absolute maximum and absolute minimum of the function $g(x, y) = x^2 - y^2$ on the region $R = \{(x, y)|x^2 + 4y^2 \leq 1\}$.

A. $\max_R g = 1$, g has no minimum

B. $\max_R g = 1, \min_R g = -\frac{1}{4}$

C. $\max_R g = 4, \min_R g = 0$

D. $\max_R g = 0, \min_R g = -\frac{1}{4}$

E. g has neither maximum nor minimum on R

F. $\max_R g = 1, \min_R g = -1$

5. $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{x^2+y^2}\, dx dy =$

A. 1   B. e^{π} C. 0

D. $\frac{1}{2}e^4$ E. $\sqrt{\pi}$ F. $\frac{\pi}{2}(e^4-1)$

6. TRUE or FALSE. For each of the following statements, indicate whether it is always true (T) or sometimes false (F). Support your answers.

   (a) Suppose $h(x, y)$ is a function of two variables so that $h_{xx}(x, y)h_{yy}(x, y) - h_{xy}(x, y)h_{yx}(x, y) \leq 0$ for all x, y in the domain of h. Then h cannot have any local maxima.

   (b) There is a point on the hyperbola $xy = 1$ for which $g(x, y) = y + \frac{1}{x}$ is minimal.

   (c) The maximum value of θ which occurs in the region bounded by $y = x$ and $y = x^2$ is $\frac{\pi}{4}$.

   (d) Let $f(r, \theta)$ be a function defined in polar coordinates, which has a local maximum at (r_0, θ_0) with $r_0 > 0$. Suppose $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ exist at (r_0, θ_0). Then $\frac{\partial f}{\partial r}|_{(r_0,\theta_0)} = 0$.