1. A coil is to be drilled out of a block of metal whose density is described by $\delta(x, y, z) = 12\sqrt{z}$. The coil to be drilled out is described by $\mathbf{r}(t) = \langle \cos t, \sin t, t^2 \rangle$, $0 \leq t \leq 2\pi$.
What is the mass of the coil?

A. 0 B. $512\pi^3$ C. $(1 + 8\pi)^{\frac{3}{2}} - 1$

D. 2π E. $4\pi^2$ F. $\sqrt{1 + 2\pi}$

2. Let $f(x, y) = x \ln(y^2 + \frac{3}{4})$. Where does $f(x, y)$ achieve local maxima?

A. $(0, \frac{1}{2})$ B. $(0, 1)$ C. $(0, 0)$

D. $(1, 1)$ E. There are no local maxima. F. $(\frac{1}{2}, 1)$

3. The plane $4x + 8(y - 1) - 2z = -6$ and the plane $2(x - 3) + y - z = 0$ intersect in a line. This line intersects the plane $x + y + z = 0$ when $z =$

A. $\frac{1}{12}$ B. 3 C. $-\frac{7}{2}$

D. 0 E. 1 F. $-\frac{13}{9}$

4. The torsion of a particle moving according to $(t^2 + t, t^2, t)$, $t \geq 0$, is:

A. $\sqrt{2t^2 + 2t^4 + 2t^3}$ B. $\sqrt{t^2 + t}$ C. 0

D. $\sqrt{t^2 - t}$ E. $(2t^2 + 2t^4)^{-\frac{3}{2}}$ F. 1

5. Assuming uniform density $\delta = 1$, the moment of the region bounded above by the sphere $x^2+y^2+z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$ about the xy-plane is:

A. 1 B. 2π C. 0

D. $\frac{5\pi}{12}$ E. $2\pi\left(\frac{1}{3}2^{\frac{3}{2}} - \frac{7}{12}\right)$ F. $\frac{\pi}{4}$

6. Which of the following planes is perpendicular to the plane which is tangent to the graph of $g(x, y) = x^2 \cos(y)$ when $(x, y) = (1, \frac{\pi}{4})$?

A. $4 = \sqrt{2}x + 3y + z$ B. $2 = x + y + \sqrt{2}z$ C. $4 = \sqrt{2}x + 3y + \sqrt{2}z$

D. $1 = \sqrt{2}x + \sqrt{2}y + z$ E. $0 = x + y + z$ F.$1 = 2x - 2y - z$

7. Integrate:

$$\int_0^2 \int_x^2 2y^2 \sin(xy) dy dx$$

A. $4 - \sin(4)$ B. 0 C. $-\cos(2)$

D. $\cos(4) - 1$ E. 4 F.$1 - \cos(2)$

8. TRUE or FALSE. For each of the following statements, indicate whether it is true (T) or false (F). Support your answers.

   i The function $g(x, y) = \begin{cases} \frac{x+y}{x-y} & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$ is continuous.

   ii The altitude of a hill is described by the function $f(x, y) = \frac{1}{2}x^2y - \pi x \cos(y)$, where x is how far east the point is and y is how far north the point is. A ball released from rest at the point on the hill corresponding to the origin will start rolling due south.

   iii Let $\mathbf{F}(x, y) = 2e^{2x+y}\mathbf{i} + e^{2x+y}\mathbf{j}$. The line integral of $\mathbf{F}$ along any closed loop is positive.

   iv The surface $\rho = \sin\phi$ is a sphere.

   v The surface $\rho = \cos\phi$ is a sphere.

   vi The graph of a solution of the differential equation $\frac{dy}{dt} = (y - 1)(y + 2)$ is a parabola.

9. Find the volume of the region which lies inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 4$.

10. Solve the initial value problem:

$$\frac{d^2\mathbf{r}}{dt^2} = -t^2\mathbf{i} + \mathbf{j} - t\mathbf{k}$$
$$\frac{d\mathbf{r}}{dt}(1) = \frac{2}{3}\mathbf{i} - \frac{1}{2}\mathbf{k}$$
$$\mathbf{r}(0) = \mathbf{i} - \mathbf{j}$$

1. C
2. E
3. F
4. C
5. D
6. D
7. A
8. i F
   ii T
   iii F
   iv F
   v T
   vi F
9. $2\pi\sqrt{5}$
10. $\mathbf{r}(t) = (-\frac{1}{12}t^4 + t + 1)\mathbf{i} + (\frac{1}{2}t^2 - t - 1)\mathbf{j} - \frac{1}{6}t^3\mathbf{k}$