MATH 114 Final Exam Practice

1. What is the angle between the curves $y = \sin(x)$ and $x = y^2$ where they intersect?

   A. 0 B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$ E. $\frac{\pi}{2}$ F. $\frac{5\pi}{6}$

2. Find the derivative of $g(x, y, z) = y\cos(x^2 + z^2)$ in the direction of the velocity vector of the curve $\mathbf{r}(t) = \langle t^2 + 2t, e^t, \sin t\rangle$ when $t = 0$, at the point described by $\mathbf{r}(0)$.

   A. 1 B. 0 C. $\sqrt{6}$ D. $\frac{1}{\sqrt{6}}$ E. $\frac{1}{2}$ F. 2

3. The minimum curvature of the ellipse $x^2 + 4y^2 = 4$ is

   A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. $\frac{1}{2}$ F. 1

4. Find $\oint_C 2x dy - 3y dx$, where C is the curve composed of a straight line segment from $(-1, 0)$ to $(0, 0)$, a straight line segment from $(0, 0)$ to $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$, and the part of the circle of radius 1, centered at the origin, traversed counterclockwise starting from $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ and ending at $(-1, 0)$.

   A. π B. $\frac{\pi}{8}$ C. $\frac{7}{8}\pi$ D. $\frac{15}{8}\pi$ E. $\frac{25}{8}\pi$ F. $\frac{35}{8}\pi$

5. Compute $\displaystyle\int_0^3 \int_{1-\frac{y}{3}}^1 \sin(x^2) dx dy$.

   A. $3\sin(1)$ B. 5 C. $5\cos(1)$ D. $\frac{3}{2} - \frac{3}{2}\cos(1)$ E. $3 - 3\sin(1)$ F. $\frac{5}{2}\cos(1)$

6. Find the integral of z over the region inside the first octant and the unit sphere, but outside the surface $\rho = \sin\phi$.

   A. 0 B. $\frac{\pi}{24}$ C. $\frac{\pi}{16}$ D. $\frac{\pi}{12}$ E. $\frac{\pi}{8}$ F. $\frac{\pi}{6}$

7. There is a unique point on the ellipsoid $x^2 + y^2 + 4z^2 = 4$ which is closest to the plane $x + y + z = 10$, and a unique point on the ellipsoid farthest from the plane. What is the distance between these two points?

   A. $\frac{2\sqrt{33}}{3}$ B. 0 C. $\sqrt{15}$ D. $\sqrt{5}$ E. $\frac{\sqrt{5}}{3}$ F. 1

8. Find and classify the critical points of the function $f(x, y) = e^{x^2} - y + xy$.

9. For which values of x, y is it true that $\langle x, 2x + 1, y\rangle \times \langle -x, y, x\rangle = \mathbf{0}$?

10. TRUE or FALSE. For each of the following statements, indicate whether it is true (T) or false (F). Support your answers.

   i. The function $g(x,y) = \begin{cases} \frac{x^2+y^2}{x+y} & \text{if } x \neq -y \\ 0 & \text{if } x = -y \end{cases}$ is continuous.

   ii. The torsion of the curve $\langle 4t, \cos(2t), \sin(2t) \rangle$ is constant.

   iii. The graph of any solution of the differential equation $\frac{dy}{dt} = y + 1$ is a line.

   iv. There is a solution of the differential equation $\frac{dy}{dt} = y^2 + 1$ whose graph is a line.

   v. The work done by the force $\mathbf{F}(x,y) = \langle y^2 e^{xy^2}, 2xye^{xy^2} + y \rangle$ to move a particle from $(0,-1)$ to $(0,1)$ along the left half of the unit circle is the the same as the work done by $\mathbf{F}$ to move a particle from $(0,-1)$ to $(0,1)$ along the right half of the unit circle.

   vi. Suppose $\mathbf{u}$ and $\mathbf{v}$ are unit vectors which are not parallel. Then $\mathbf{u} \times \mathbf{v}$ is a unit vector.

1. E
2. D
3. C
4. F
5. D
6. B
7. A
8. Saddle point at $(1, -2e)$.
9. $(x, y) = (-1, 1)$ or $(x, y) = (0, 0)$.
10.
    i. F
    ii. T
    iii. F
    iv. F
    v. T
    vi. F