

1. What is the angle between the curves  $y = \cos(x - 1) - 1$  and  $x = e^y$  where they intersect?

- A. 0   B.  $\frac{\pi}{6}$    C.  $\frac{\pi}{4}$    D.  $\frac{\pi}{3}$    E.  $\frac{\pi}{2}$    F.  $\frac{5\pi}{6}$

2. Find the moment of inertia about the  $z$ -axis of the portion of the first octant inside the surface  $\rho = 2 \sin \phi$ , if density is given by  $\delta(x, y, z) = z$ .

- A. 0   B.  $\frac{\pi}{5}$    C.  $\frac{3\pi}{5}$    D.  $\frac{6\pi}{5}$    E.  $\frac{8\pi}{5}$    F.  $\frac{11\pi}{5}$

3. Suppose  $\frac{dy}{dt} = k(y + 7)$ ,  $y(0) = 0$ , and  $y(1) = 1$ . What is  $y(2)$ ? Here  $k$  is a constant.

- A.  $\frac{15}{7}$    B.  $\frac{13}{7}$    C.  $\frac{11}{7}$    D.  $\frac{9}{7}$    E. 1   F.  $\frac{5}{7}$

4. What is the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(3x^2 + y^2)(1 - \cos(x^2 + y^2))}{x^6 + y^6}$$

- A.  $\frac{3}{2}$    B.  $\frac{5}{2}$    C.  $\frac{7}{2}$    D.  $\frac{9}{2}$    E.  $\frac{11}{2}$    F. limit does not exist

5. The maximal volume of any rectangular prism whose sides  $a, b, c$  satisfy  $4ab + 9ac + bc \leq 108$  is:

- A. 36   B.  $\frac{71}{2}$    C. 35   D. 34   E. 33   F.  $\frac{65}{2}$

6. Consider a parallelogram  $\mathcal{P}$ , three of whose vertices are  $A = (1, 2, 1)$ ,  $B = (0, 1, 1)$ ,  $C = (-1, 0, 0)$ .

- Find the area of  $\mathcal{P}$ .
- $\mathcal{P}$  lies in a plane. Give an equation for that plane.
- If we project  $\mathcal{P}$  to the  $yz$  plane, we get a parallelogram in the  $yz$  plane. What is the area of that parallelogram?

7. Suppose  $R$  is the parallelepiped spanned by  $\mathbf{u} = \langle a, 0, 0 \rangle$ ,  $\mathbf{v} = \langle b_1, b_2, 0 \rangle$ ,  $\mathbf{w} = \langle c_1, c_2, c_3 \rangle$ .

- Give a formula for the surface area  $A$  in terms of  $a, b_1, b_2, c_1, c_2, c_3$ .
- Give a formula for the volume  $V$  in terms of  $a, b_1, b_2, c_1, c_2, c_3$ .
- If we wanted to find the parallelepiped with maximal volume among all such parallelepipeds with the same surface area  $A_0$ , write (but don't solve) the Lagrange multipliers system in the variables  $a, b_1, b_2, c_1, c_2, c_3$  that we'd need to solve.
- By examining the equations in the Lagrange multipliers system, show that  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  must be mutually orthogonal to maximise volume. (*Hint.* Consider  $\frac{\partial}{\partial b_1} V = \lambda \frac{\partial}{\partial b_1} A$ .)

8. TRUE or FALSE. For each of the following statements, indicate whether it is true (T) or false (F). Support your answers.

- The graph of a solution of the differential equation  $\frac{dy}{dt} = \cos y$  is a sinusoidal wave.
- The graph of a solution of the differential equation  $\frac{dy}{dt} = \sin t$  is a sinusoidal wave.
- The curve  $\mathbf{r}(t) = \langle \frac{1}{4}t^3 - 2, \frac{4}{t} - 3, \cos(t - 2) \rangle$  is tangent to the surface  $x^3 + y^3 + z^3 - xyz = 0$  at the point  $(0, -1, 1)$ .