# MTH 132.12 Final Exam Review

Answer ten of the following questions on a separate sheet for your quiz 12 grade. You **must** answer questions 7, 8, 9, and 14. Due **Wednesday, 27 April**.

1. Find the following limits, or show that they do not exist.
   (a) $\lim_{x\to 1} \dfrac{\sin^2(x-1)}{(x-1)}$
   (b) $\lim_{x\to 0^+} \dfrac{|x|}{x^2}$
   (c) $\lim_{t\to\infty} \dfrac{\cos(t)}{t}$
   (d) $\lim_{t\to 0} \dfrac{\tan(t)}{2t}$

2. Show, using the formal definition of limit, that $\lim_{t\to 9} \sqrt{t+7} = 4$.

3. Establish, using the definition of the derivative, that $\dfrac{d}{ds}|_{s=5}(s^2+s+1) = 11$.

4. Where is the function $f(x) = \frac{|\sin(x)|}{\cos(x)}$ continuous? Where is it differentiable?

5. Is the function $g(s) = \begin{cases} s & -1 \le s < 0 \\ \sin(s) & 0 \le s \le \pi \end{cases}$ continuous at $s = 0$? Is it differentiable at $s = 0$? Explain.

6. Differentiate the following functions:
   (a) $z = \dfrac{1}{\sqrt{t}+1}$
   (b) $u = \left(\dfrac{\tan\theta}{1+\sin\theta}\right)^2$
   (c) $y = \tan(\cos(x))$
   (d) $\ell = \tan(x)\cos(x)$
   (e) $h = \displaystyle\int_1^t \sin(q)dq$
   (f) $h = \displaystyle\int_t^5 \frac{2z}{z^2+1}dz$

7. Find the following indefinite integrals.
   (a) $\int (t - \frac{4}{t})(t + \frac{4}{t})dt$
   (b) $\int \sec(t)\tan(t)\sqrt{1 + 3\sec(t)}dt$
   (c) $\int \cos^2(z)dz$

8. Let $f$ be a function which has a positive derivative everywhere, and $f(1) = 0$. Answer the following questions about $g(t) = \int_0^t f(x)dx$.
   (a) Is $g$ differentiable?
   (b) Is $g$ is continuous?
   (c) Give a formula for the tangent line to the graph of $g$ at $t = 1$.
   (d) What is the concavity of the graph of $g$?
   (e) What are the critical points of $g$?
   (f) What are the inflection points of $g$?

9. What is the average value of the function $g(z) = |\sin(z)\cos(z)|$ on the interval $[0, 2\pi]$?

10. Find a function $y$ which satsifies $\dfrac{dy}{dx} = 3\sin^2(x) + 2$ and $y(\pi) = \frac{\pi}{2}$.

11. Set up, but do not evaluate, the Riemann sum with $n = 5$, using the right-hand method, which estimates $\int_1^6 (x^2 + 1)dx$. Use $\Sigma$-notation.

12. Find the height and radius of the largest (i.e. with the most volume) cylinder which can be fit inside a sphere of radius $\sqrt{3}$.

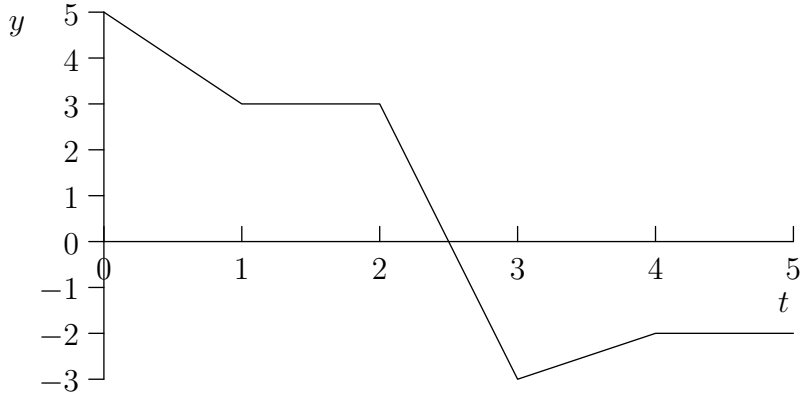
13. Find the tangent and normal lines to the graph of the relation $y = \sqrt{xy}$ at the point $(1, 1)$.

14. Let $h(z) = z^2 + z - 6$.
   (a) What is the signed area under the graph of $h$ from $z = -4$ to $z = 4$?
   (b) What is the total area between the graph of $h$ and the $z$-axis, from $z = -4$ to $z = 4$? (*Hint.* You'll need to find the zeroes of $h$.)

15. Find the local and global extreme values of the function $y(x) = -x^2 + 2\sqrt{x}$ on the interval $[0, \sqrt[3]{4}]$.

16. Let $f(x)$ be a continuous function. Suppose that $0 \leq f(x) \leq 1$ for all $x$ in the interval $[0, 1]$. Show that there is some $c$ in $[0, 1]$ such that $f(c) = c$.

17. Suppose the graph below is my velocity in miles per hour.




(a) What is my total change in position after 5 hours?

(b) What is the total distance I have walked in 5 hours?

18. Show that for any numbers $a, b$, $|\cos(b) - \cos(a)| \leq |b - a|$. (*Hint.* Use a Theorem and $|\sin(x)| \leq 1$.)

19. The following table gives information about the functions $f$ and $g$.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| -1 | 1 | 1 | 0 | 3 |
| 0 | 0 | 2 | 0 | 0 |
| 1 | 2 | 3 | 2 | 0 |
| 2 | 3 | -1 | -1 | 2 |
| 3 | -1 | 0 | 3 | -1 |

(a) $\frac{d}{dx}|_{x=1}(f(x) + g(x)) =$

(b) $\frac{d}{dx}|_{x=0}(f(x)g(x)) =$

(c) $\frac{d}{dx}|_{x=2}(\frac{f(x)}{g(x)}) =$

(d) $\frac{d}{dx}|_{x=1}(f(g(x))) =$

(e) $\frac{d}{dx}|_{x=1}(g(f(x))) =$

(f) $\frac{d}{dx}|_{x=-1}((f(x))^2) =$