MTH 132.12 Quiz 10
Friday 8 April 2011

Name:

Show *all* your work. Points will be deducted for incomplete work. No calculators are allowed.

1. Write the following sums using Σ notation.

   (a) $2 + 5 + 8 + 11 + 14 + 17 + 20$

   $$\sum_{k=0}^{6}(3k+2) \text{ or } \sum_{k=1}^{7}(3k-1)$$

   (b) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

   $$\sum_{k=0}^{6}\frac{1}{k+2} \text{ or } \sum_{k=1}^{7}\frac{1}{k+1} \text{ or } \sum_{k=2}^{8}\frac{1}{k}$$

2. Suppose $\sum_{k=1}^{n} a_k = 2$ and $\sum_{k=1}^{n} b_k = 3$.

   (a) What is $\sum_{k=1}^{n}(2a_k + b_k)$?

   $$\sum_{k=1}^{n}(2a_k + b_k) = \sum_{k=1}^{n} 2a_k + \sum_{k=1}^{n} b_k$$
   $$= 2\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$
   $$= 2 \cdot 2 + 3 = 7$$

   (b) What is $\sum_{k=1}^{n}(b_k + 1)$?

   $$\sum_{k=1}^{n}(b_k + 1) = \sum_{k=1}^{n} b_k + \sum_{k=1}^{n} 1$$
   $$= 3 + n$$

3. What is $\int_{1}^{5} \sqrt{16 - (x-1)^2}\,dx$? (*Hint.* The graph $y = \sqrt{16 - (x-1)^2}$ is a familiar shape.)

   $\int_{1}^{5} \sqrt{16 - (x-1)^2}\,dx$ is the area between the x-axis, the line $x = 1$, and the upper half of the circle $(x-1)^2 + y^2 = 16$. Thus it is the area of one-quarter of a disc of radius 4.

   So $\int_{1}^{5} \sqrt{16 - (x-1)^2}\,dx = \frac{1}{4}\pi 4^2 = 4\pi$.