MTH 132.12 Quiz 2
Friday 28 January 2011

Name:

Show *all* your work. Points will be deducted for incomplete work. No calculators are allowed.

1. For each statement below, indicate whether it is true or false. Give a short explanation.

   (a) $\lim_{x\to 0} \frac{1}{x} = \infty$

   False. $\lim_{x\to 0^+} \frac{1}{x} = \infty$ but $\lim_{x\to 0^-} \frac{1}{x} = -\infty$

   (b) $\lim_{x\to 1} \frac{\sin(x^2-1)}{x^2-1} = 1$

   True. $\lim_{x\to 1} x^2 - 1 = 0$, so we can use the fact that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$.

   (c) $\lim_{x\to \infty} \frac{\sin(x^2-1)}{x^2-1} = 1$

   False. The numerator is bounded by 1 and -1, but the numerator is growing without bound. Hence the limit is 0, not 1.

2. Consider the function $h(t) = \begin{cases} \frac{t^2-3t-4}{(t-4)(t+3)} & \text{if } t \neq 4, -3 \\ 12 & \text{if } t = 4 \\ 1 & \text{if } t = -3 \end{cases}$

   (a) What are the discontinuities of $h(t)$? Show your work.

   $h(t)$ is continuous at every point except $t = -3$ and $t = 4$, since it is a rational function with nonzero denominator. Compute

   $$\begin{aligned} \lim_{t\to 4} h(t) &= \lim_{t\to 4} \frac{t^2-3t-4}{(t-4)(t+3)} \\ &= \lim_{t\to 4} \frac{t+1}{t+3} \\ &= \frac{5}{7} \end{aligned}$$

   Since this limit is not equal to $h(4)$, h is not continuous at $t = 4$.

   On the other hand, at $t = -3$, $h(t)$ exhibits unbounded growth, hence is not continuous.

   (b) Which of the discontinuities of $h(t)$ are removable? Explain.

   If we define $H(t) = \begin{cases} h(t) & \text{if } t \neq 4 \\ \frac{5}{7} & \text{if } t = 4 \end{cases}$, then $H(t)$ is continuous at $t = 4$, and $H(t)$ is a continuous extension of $h(t)$. So the discontinuity of $h(t)$ at $t = 4$ is removable.
   On the other hand, $h(t)$ exhibits unbounded growth near $t = -3$, so the discontinuity there is *not* removable.