

MTH 132.12 Quiz 4
Friday 18 February 2011

Name:

Show *all* your work. Points will be deducted for incomplete work. No calculators are allowed.

1. Compute the following derivatives. Show each derivative rule or formula you use.

(a) $\frac{d}{dx} \sin(x^2 + 2x)$

$$\frac{d}{dx} \sin(x^2 + 2x) = \cos(x^2 + 2x) \frac{d}{dx} (x^2 + 2x) = (2x + 2) \cos(x^2 + 2x)$$

(b) $\frac{d}{dx} \left[\frac{(6x - 9)(4x + 6)}{8x^2 - 18} \right]$

$$\frac{d}{dx} \left[\frac{(6x - 9)(4x + 6)}{8x^2 - 18} \right] = \frac{d}{dx} \left[\frac{3(2x - 3)2(2x + 3)}{2(4x^2 - 9)} \right] = \frac{d}{dx} \left[\frac{6(4x^2 - 9)}{2(4x^2 - 9)} \right] = \frac{d}{dx} [3] = 0$$

Alternatively,

$$\begin{aligned} \frac{d}{dx} \left[\frac{(6x - 9)(4x + 6)}{8x^2 - 18} \right] &= \frac{d}{dx} \left[\frac{24x^2 - 54}{8x^2 - 18} \right] \\ &= \frac{(8x^2 - 18) \frac{d}{dx} [24x^2 - 54] - (24x^2 - 54) \frac{d}{dx} [8x^2 - 18]}{(8x^2 - 18)^2} \\ &= \frac{(8x^2 - 18)48x - (24x^2 - 54)16x}{(8x^2 - 18)^2} \\ &= \frac{384x^3 - 864x - (384x^3 - 864x)}{(8x^2 - 18)^2} \\ &= \frac{0}{(8x^2 - 18)^2} = 0 \end{aligned}$$

2. Suppose f and g are differentiable functions with the following values:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	4	1	$\frac{1}{5}$
1	3	2	0	$\frac{1}{3}$

Compute the following derivatives. Show each step.

(a) $\frac{d}{dx} \Big|_{x=0} [f(g(x))]$

$$\frac{d}{dx} \Big|_{x=0} [f(g(x))] = f'(g(0))g'(0) = f'(1)g'(0) = 2 \cdot \frac{1}{5} = \frac{2}{5}$$

(b) $\frac{d}{dx} \Big|_{x=1} \left[\frac{f(x)}{g(x) + 1} \right]$

$$\begin{aligned} \frac{d}{dx} \Big|_{x=1} \left[\frac{f(x)}{g(x) + 1} \right] &= \frac{(g(1) + 1)f'(1) - f(1) \frac{d}{dx} \Big|_{x=1} (g(x) + 1)}{(g(1) + 1)^2} \\ &= \frac{(g(1) + 1)f'(1) - f(1)g'(1)}{(g(1) + 1)^2} \\ &= \frac{(0 + 1) \cdot 2 - 3 \cdot \frac{1}{3}}{(0 + 1)^2} = 1 \end{aligned}$$