MTH 132.12 Quiz 5
Friday 25 February 2011

Name:

Show *all* your work. Points will be deducted for incomplete work. No calculators are allowed.

1. Consider the circles $C_1 = \{(x,y)|x^2+y^2=1\}$ and $C_2 = \{(x,y)|(x-\sqrt{2})^2+y^2=1\}$.
   (a) What are the slopes of C_1 and C_2, respectively, at $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$?

   For C_1, we have by implicit differentiation

   $$2x + 2y\frac{dy}{dx} = 0$$
   $$\frac{dy}{dx} = -\frac{x}{y}$$

   So the slope of C_1 at $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ is $-\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$.

   For C_2, we have by implicit differentiation

   $$2(x-\sqrt{2}) + 2y\frac{dy}{dx} = 0$$
   $$\frac{dy}{dx} = -\frac{x-\sqrt{2}}{y}$$

   So the slope of C_2 at $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ is $-\frac{\frac{\sqrt{2}}{2}-\sqrt{2}}{\frac{\sqrt{2}}{2}} = -\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$.

   (b) What can you say about the way the two circles intersect?

   The two curves meet at right angles at the point $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$.

2. Suppose a particle is moving, with velocity given by $v(t) = -\frac{2}{t+3}$ for $t \geq 0$. Velocity is measured in furlongs per day (fur/day) and time is measured in days.
   (a) What is the acceleration $a(t)$?

   $$a(t) = \frac{d}{dt}(-\frac{2}{t+3}) = \frac{d}{dt}(-2(t+3)^{-1}) = (-1)(-2)(t+3)^{-2}\frac{d}{dt}(t+3) = 2(t+3)^{-2} = \frac{2}{(t+3)^2}$$

   (b) What are the units of $a(t)$?

   furlongs per day per day, or $\frac{\text{fur}}{\text{day}^2}$

   (c) True or false: the particle is acted on by a constant force (i.e. the force acting on the particle does not change over time).

   False. The acceleration changes in t.

   (d) When does the particle change direction?

   The velocity $v(t) = -\frac{2}{t+3}$ is always negative. Thus the particle never changes direction.