MTH 132.12 Quiz 6

Due Monday 14 March 2011

Name:

The linearization of $f(x)$ at $x = c$ is the linear function whose graph is the tangent line to the graph $y = f(x)$ at $x = c$. That is, the linearization is $L(x) = f'(c)(x - c) + f(c)$. In particular we have $L(c) = f(c)$ and $L'(c) = f'(c)$. Because the graph and its tangent line are close near $x = c$, we can use L to estimate values of f near c.

Sometimes the linearization is a good approximation, for example the function $f(x) = \sin x$ near $x = 0$. But sometimes the linear approximation is not as good, for example the function $f(x) = \cos x$ near $x = 0$. In fact $\cos x$ looks somewhat like a parabola near $x = 0$, so we might ask if we can use a parabola to approximate its graph instead of a line.

We say that $Q(x)$ is *the quadratic approximation* to $f(x)$ at $x = c$ if $Q(x)$ is a quadratic function such that:

- $Q(c) = f(c)$
- $Q'(c) = f'(c)$
- $Q''(c) = f''(c)$

Answer the following questions about quadratic approximation. Be sure to show your work.

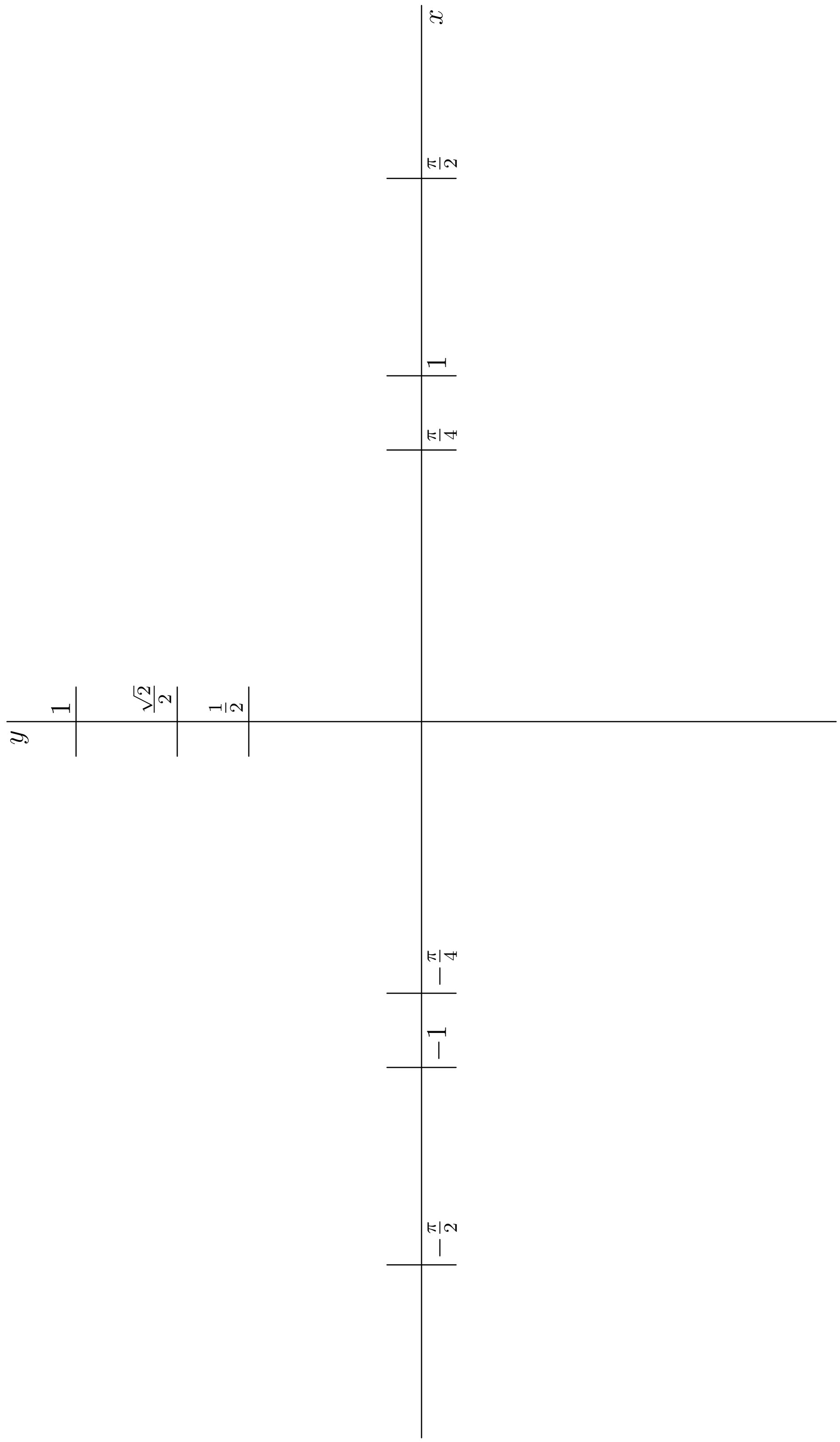
1. Given any number c, any quadratic function $Q(x)$ can be written as $Q(x) = b_0 + b_1(x-c) + b_2(x-c)^2$. If $Q(x)$ is the quadratic approximation to a function $f(x)$ at $x = c$, what are b_0, b_1, and b_2?

2. Let $L(x)$ be the linear approximation and $Q(x)$ the quadratic approximation to $\cos x$ at $x = 0$.
   (a) Give formulae for $L(x)$ and $Q(x)$.

   (b) By hand, compute $L(\frac{1}{2})$ and $Q(\frac{1}{2})$.

   (c) Use a calculator to give a three-place decimal approximation to $\cos(\frac{1}{2})$.

   (d) Carefully graph $L(x)$, $Q(x)$, and $\cos x$ on the axes on the next page.



x
$\frac{\pi}{2}$
1
$\frac{\pi}{4}$
$-\frac{\pi}{4}$
-1
$-\frac{\pi}{2}$
y
1
$\frac{\sqrt{2}}{2}$
$\frac{1}{2}$