MTH 132.12 Quiz 7
Friday 18 March 2011

Name:

Show *all* your work. Points will be deducted for incomplete work. No calculators are allowed.

Let $g(x) = (1-x^2)^{\frac{1}{3}}$.

1. What is $g'(x)$?

$$\begin{aligned}\frac{d}{dx}g(x) &= \frac{1}{3}(1-x^2)^{-\frac{2}{3}}\frac{d}{dx}(1-x^2)\\ &= \frac{-2x}{3(1-x^2)^{\frac{2}{3}}}\end{aligned}$$

2. Find the critical points of $g(x)$.

$g'(x)$ does not exist when $x=1$ or $x=-1$, because the denominator is zero there. Solving the equation $g'(x)=0$,

$$\begin{aligned}\frac{-2x}{3(1-x^2)^{\frac{2}{3}}} &= 0\\ -\frac{2}{3}x &= 0(1-x^2)^{\frac{2}{3}} = 0\\ x &= 0\end{aligned}$$

So the critical points are $\{-1, 0, 1\}$.

3. For each critical point, say whether it is where g achieves a local maximum, a local minimum, or neither. Explain your reasoning.

First try the second derivative test. Compute:

$$\begin{aligned}g''(x) &= \frac{3(1-x^2)^{\frac{2}{3}}(-2) - (-2x)\frac{2(-2x)3}{3(1-x^2)^{\frac{1}{3}}}}{9(1-x^2)^{\frac{4}{3}}}\\ &= \frac{-6(1-x^2)^{\frac{2}{3}} - \frac{8x^2}{(1-x^2)^{\frac{1}{3}}}}{9(1-x^2)^{\frac{4}{3}}}\end{aligned}$$

$g''(0) = \frac{-6}{9} = -\frac{2}{3} < 0$, so g has a local maximum at $x=0$.
The second derivative test won't work at $x=\pm 1$ since g'' is not defined there. So use the first derivative test. Since $(1-x^2)^{\frac{2}{3}}$ is always positive, $g'(x)$ has the opposite sign from x. So $g'(x)$ is positive whenever x is negative. In particular, $g'(x)$ does not change sign at $x=-1$. Similarly, $g'(x)$ does not change sign at $x=1$. So the critical points $x=1$, $x=-1$ are not where g achieves any local extreme.