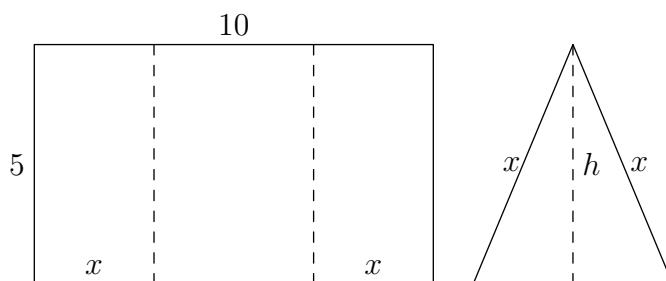


MTH 132.12 Quiz 9
Friday 1 April 2011

Name:

Show *all* your work. Points will be deducted for incomplete work. No calculators are allowed.

I have a 5-by-10-inch rectangle of paper. I want to fold the paper at two places along the 10-inch side to make a triangular prism. See the figures below.



1. Find h in terms of x .

The base of the triangle is $10 - 2x$. So we have $(5 - x)^2 + h^2 = x^2$, or $h = \sqrt{10x - 25}$.

2. Write a formula for the area of the cross-section in terms of x .

From above, $A = \frac{1}{2}(10 - 2x)\sqrt{10x - 25}$.

3. How far in from the edge of the paper should I make my folds, in order to maximize the volume of the resulting prism?

$$V(x) = 5A = 5\frac{1}{2}(10 - 2x)\sqrt{10x - 25} = 5(5 - x)\sqrt{10x - 25}$$

So differentiating,

$$V'(x) = 5 \left[(5 - x) \frac{5}{\sqrt{10x - 25}} - \sqrt{10x - 25} \right]$$

This derivative does not exist when $x = 2.5$. It is zero at the solutions of:

$$\begin{aligned} \frac{25 - 5x}{\sqrt{10x - 25}} &= \sqrt{10x - 25} \\ 25 - 5x &= 10x - 25 \\ 50 &= 15x \\ \frac{10}{3} &= x \end{aligned}$$

The critical point $x = 2.5$ gives $V(2.5) = 0$, so we know that $x = \frac{10}{3}$ is the critical point we are looking for.