Subsets of the Plane
textbook section 17.3

MATH 241

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Circles and Discs

The set $\{z||z - z_0| = \rho\}$ is the circle of radius ρ and centre z_0.

The set $\{z||z - z_0| < \rho\}$ is the ρ-neighbourhood of z_0 or the open disc of radius ρ and centre z_0.

Definition

A set S of complex numbers is open if, for every z in S, there is a ρ so that the ρ-neighbourhood of z_0 lies entirely inside S.

Definition

A point z with the property that every ρ-neighbourhood of z_0 contains some point of S and some point outside of S is a boundary point for S.

The set of all boundary points of S is called the boundary of S.

Definition

A set which contains its boundary is called closed.

rough heuristic

Sets with definitions involving $<$ are probably open; sets with definitions involving $\leq$ are probably closed.

Definition

The set S is connected if any two points z_1, z_2 in S can be joined by a (polygonal) curve lying entirely inside S.

Definition

A domain is a connected open set.

A region is a domain along with some of its boundary points.