Cauchy's Formulae
textbook section 18.4

MATH 241

March 29, 2012

Theorem (Cauchy-Goursat)

If f is analytic in a domain D, then for purposes of integrating f, we can deform contours across D.

Computation

$\oint_C \frac{dz}{z-z_0} = 2\pi i$, where C is any simple closed contour around z_0.

Note

If $f(z)$ is analytic in D and z_0 is a point of D, then $\frac{f(z)}{z-z_0}$ is analytic everywhere in D except at z_0.

Cauchy's Formula 0

If D is a simply-connected domain, f is analytic in D, and z_0 is a point of D, then

$$\oint_C \frac{f(z)}{z - z_0} dz = 2\pi i f(z_0)$$

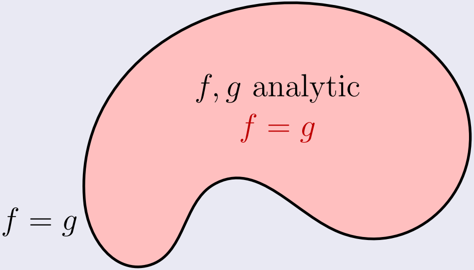
for any simple closed contour C around z_0.

We could also write

$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0} dz$$

Theorem

If f and g are both analytic in the simply-connected domain D, and $f(z) = g(z)$ for all z in some simple closed contour C, then $f(z) = g(z)$ for all z inside C.



Cauchy's Formula 1

If D is a simply-connected domain, f is analytic in D, and z_0 is a point of D, then

$$\oint_C \frac{f(z)}{(z-z_0)^2}\,dz = 2\pi i f'(z_0)$$

for any simple closed contour C around z_0.

Cauchy's Formula in general

If D is a simply-connected domain, f is analytic in D, and z_0 is a point of D, then

$$\oint_C \frac{f(z)}{(z-z_0)^{n+1}}\,dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$$

for any simple closed contour C around z_0.

Remarkable Fact

If D is a simply-connected domain and f is analytic in D, then f has derivatives of all orders in D.

This is completely different from real functions!