

Series Representations II: Fourier Series textbook section 12.2

MATH 241

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The set

$$\left\{1, \cos\left(\frac{\pi}{p}x\right), \cos\left(\frac{2\pi}{p}x\right), \dots, \sin\left(\frac{\pi}{p}x\right), \sin\left(\frac{2\pi}{p}x\right), \dots\right\}$$

is orthogonal on the interval $[-p, p]$.

In fact, this set is **complete**.

This set is known as the **Fourier basis** on $[-p, p]$.

Fourier coefficients

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right) \right)$$

where

$$a_0 = \frac{1}{p} \int_{-p}^p f(t) dt$$

$$a_n = \frac{1}{p} \int_{-p}^p f(t) \cos\left(\frac{n\pi}{p}t\right) dt$$

$$b_n = \frac{1}{p} \int_{-p}^p f(t) \sin\left(\frac{n\pi}{p}t\right) dt$$















