Series Representations III: Fourier Series textbook section 12.2

MATH 241

January 17, 2012

Fourier coefficients

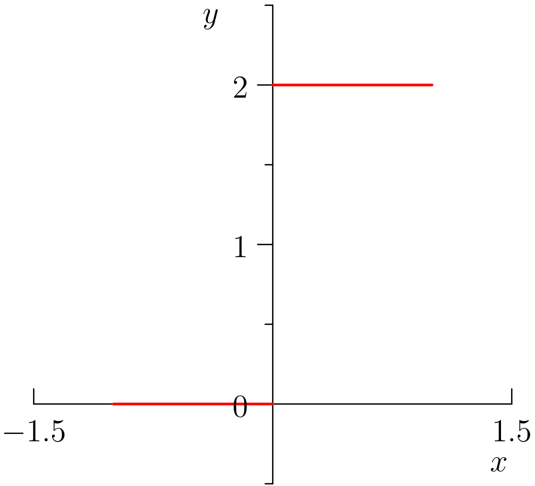
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)$$

where

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(t)dt$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(t)\cos\left(\frac{n\pi}{p}t\right) dt$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(t)\sin\left(\frac{n\pi}{p}t\right) dt$$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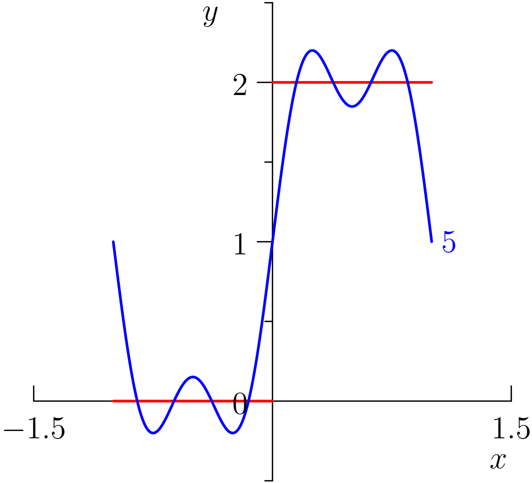
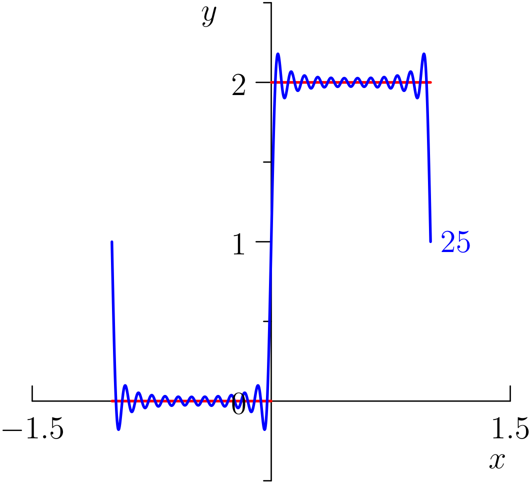
2
1
0
−1.5
1.5
x


y
2
1
0
5
−1.5
1.5
x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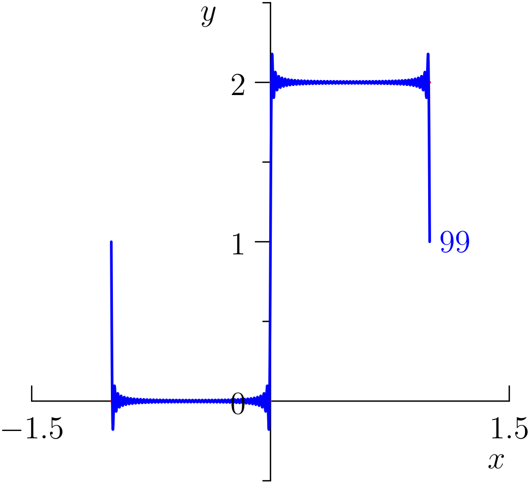
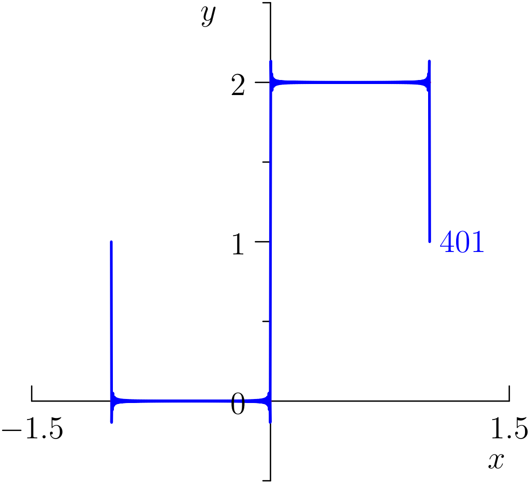
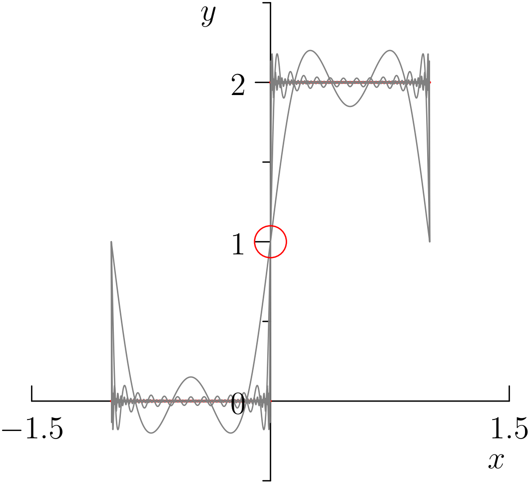
2
1
0
25
−1.5
1.5
x


y
2
1
0
99
−1.5
1.5
x



y
2
1
0
401
−1.5
1.5
x


y
2
1
0
−1.5
1.5
x

Theorem

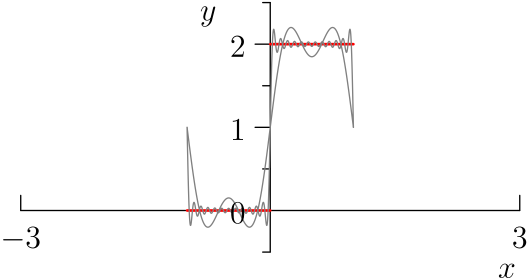
If f is a function which is piecewise differentiable on $(-p, p)$ and f' is piecewise continuous on $(-p, p)$, then at any point x in the interval $(-p, p)$ where f is continuous, the Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right) \right)$$

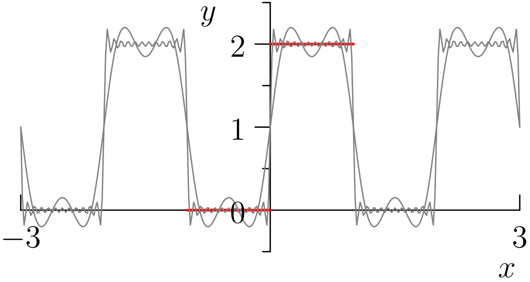
converges to $f(x)$.
At any point x where f or f' is not continuous, the Fourier series converges to

$$\frac{\lim_{t\to x^+} f(t) + \lim_{t\to x^-} f(t)}{2}$$

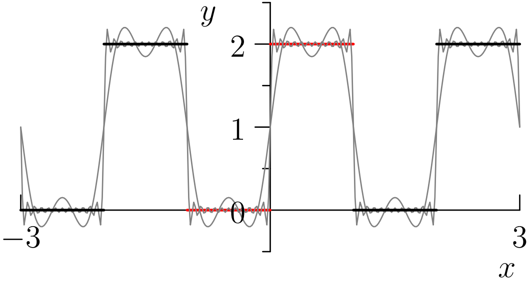
the average of the two handed limits of f at x.



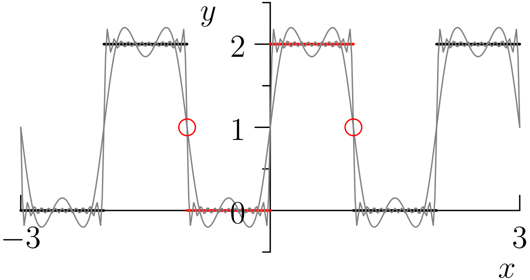
y
2
1
0
−3
3
x



y
2
1
0
−3
3
x



y
2
1
0
−3
3
x



y
2
1
0
−3
3
x

Theorem

If f is a function which is piecewise differentiable on $(-p, p)$ and f' is piecewise continuous on $(-p, p)$, then the Fourier series converges to:

$$\begin{cases} f(x) \\ \frac{1}{2}\left(\lim_{t\to x^+} f(t) + \lim_{t\to x^-} f(t)\right) \\ \frac{1}{2}\left(\lim_{t\to -p^+} f(t) + \lim_{t\to p^-} f(t)\right) \end{cases}$$