Series Representations IV: Sine and Cosine Series textbook section 12.3

MATH 241

January 17, 2012

Definition

$f(x)$ is an odd function if $f(-x) = -f(x)$.
$f(x)$ is an even function if $f(-x) = f(x)$.

Facts About Even and Odd Functions

$\cdot$	*even*	*odd*
even	*even*	*odd*
odd	*odd*	*even*

- $$\int_{-a}^{a} f(x)dx = \begin{cases} 0 & \text{if } f \text{ is odd} \\ 2\int_{0}^{a} f(x)dx & \text{if } f \text{ is even} \end{cases}$$

Fourier Series for Odd Functions

If f is an odd function,

$$a_m = 0$$
$$b_m = \frac{2}{p}\int_0^p f(x)\sin\left(\frac{m\pi}{p}x\right)dx$$

We call this a sine series.

Fourier Series for Even Functions

If f is an even function,

$$a_m = \frac{2}{p}\int_0^p f(x)\cos\left(\frac{m\pi}{p}x\right)dx$$
$$b_m = 0$$

We call this a cosine series.

Even Extension Cosine ("Half-Range Cosine") Series

For f defined on $[0, L]$, the series

$$\frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos\left(\frac{m\pi}{L}x\right)$$

converges* to $f(x)$, where

$$a_m = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right)dx$$

Odd Extension Sine ("Half-Range Sine") Series

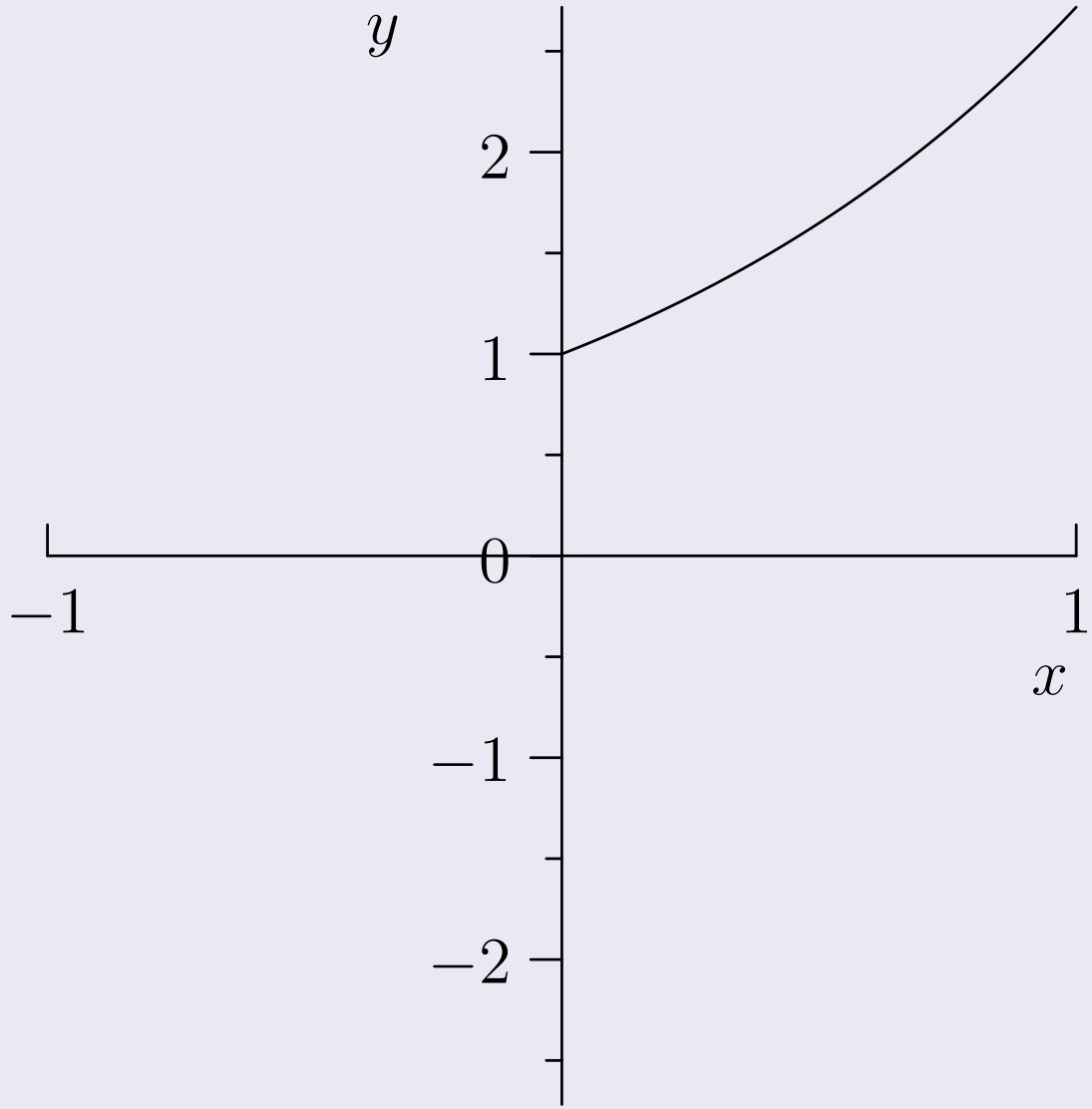
For f defined on $[0, L]$, the series

$$\sum_{m=1}^{\infty} b_m \sin\left(\frac{m\pi}{L}x\right)$$

converges* to $f(x)$, where

$$b_m = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx$$

Cosine Series for $e^x, 0 \leq x \leq 1$



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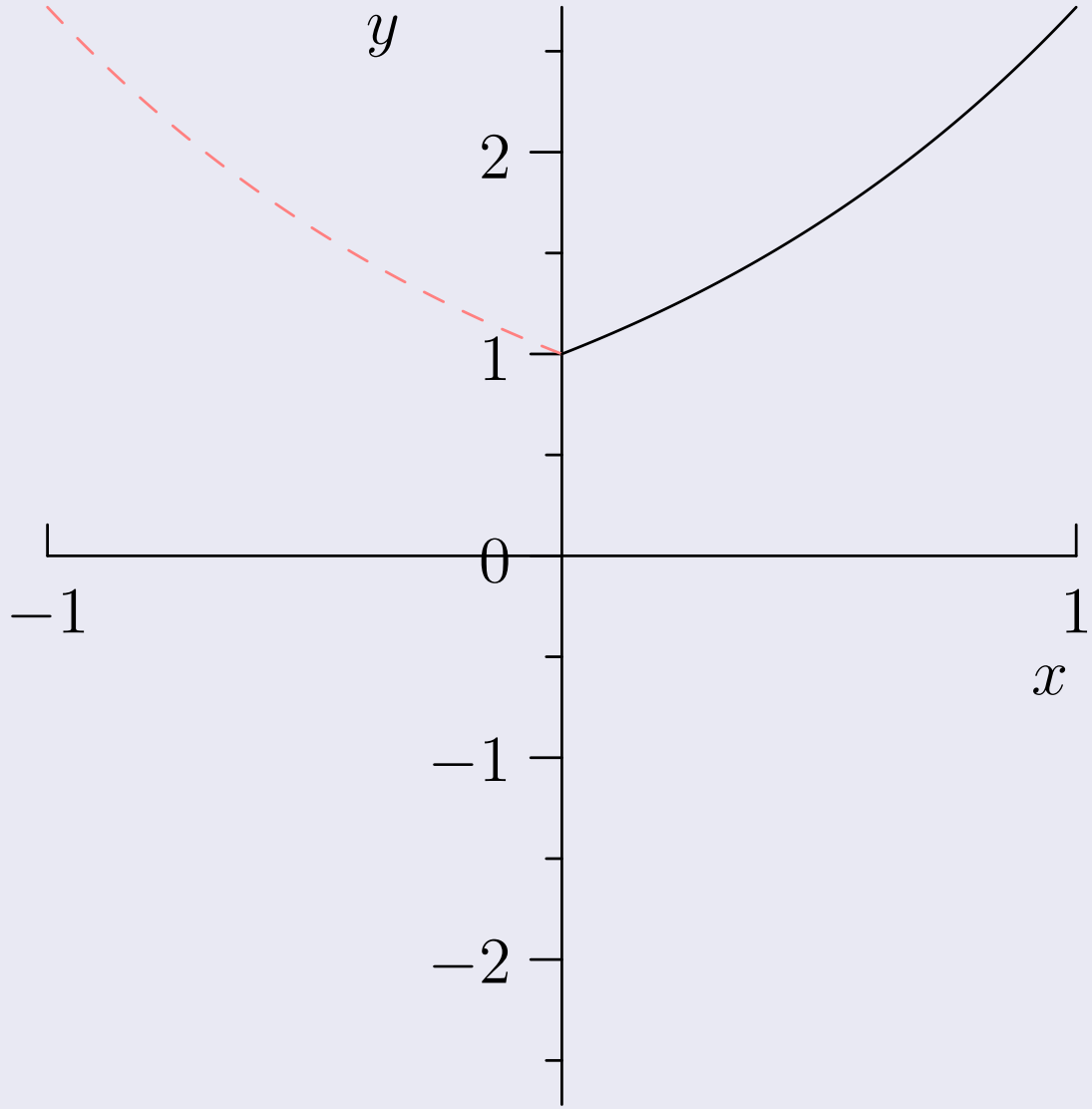

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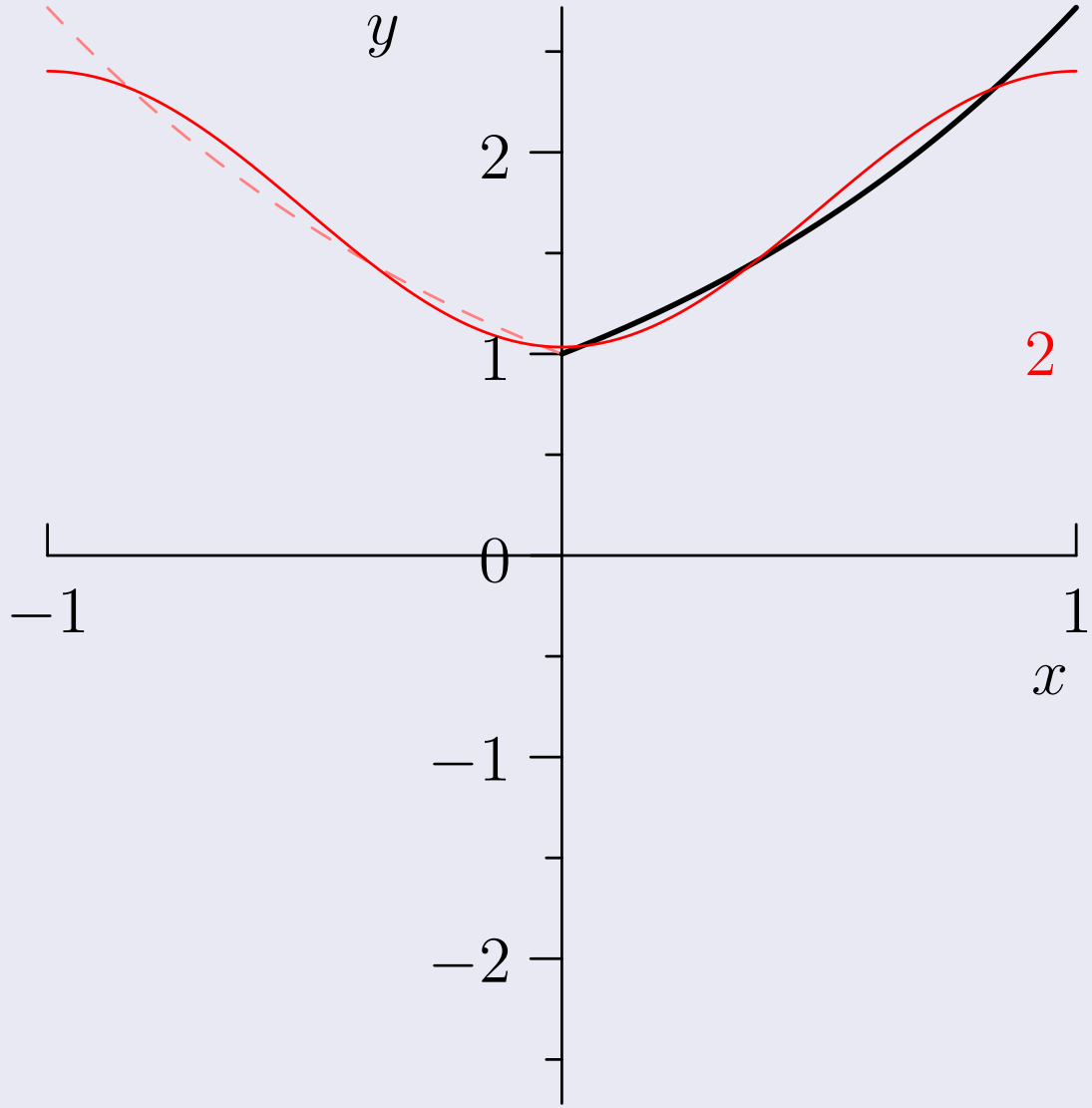

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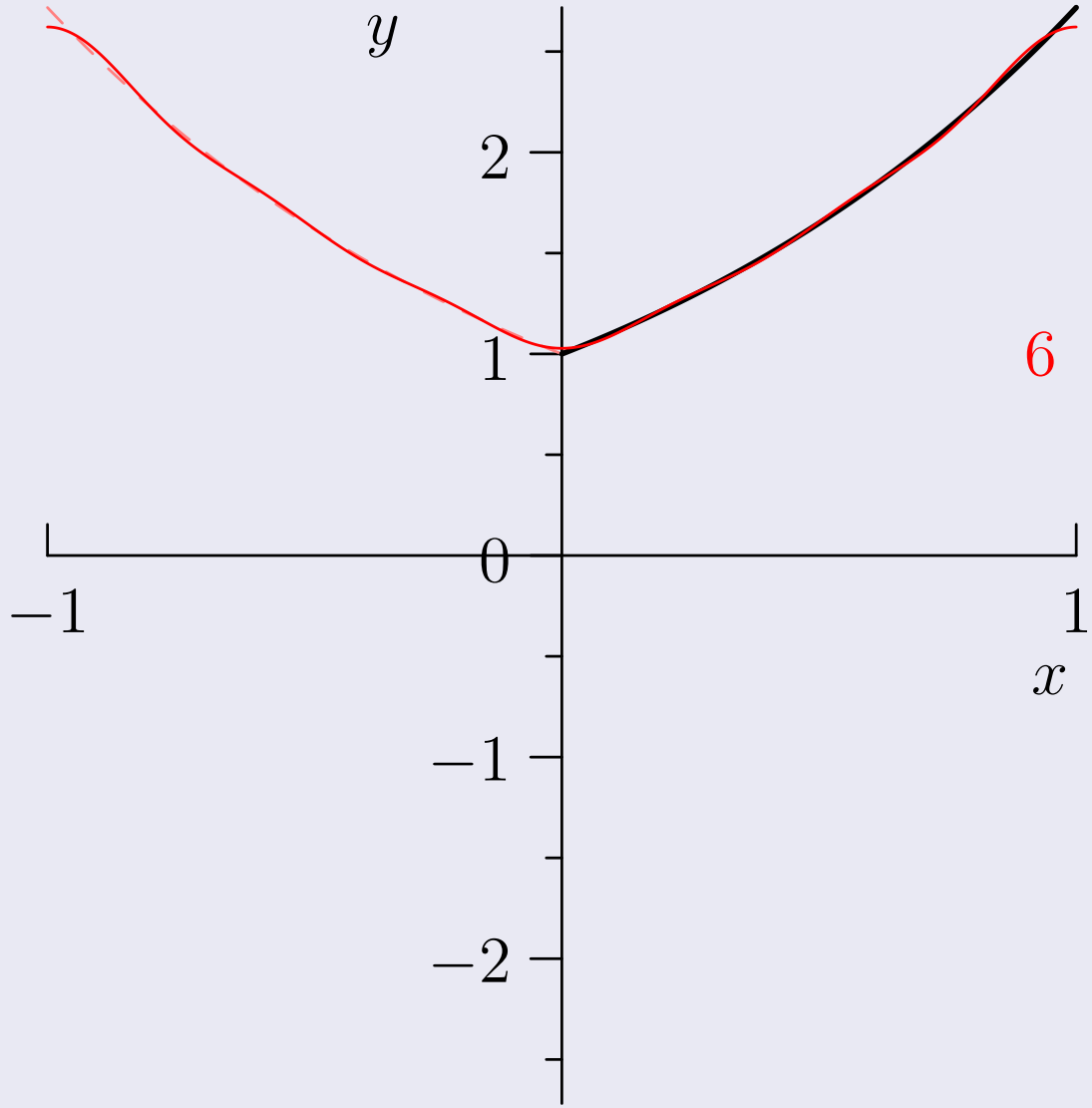

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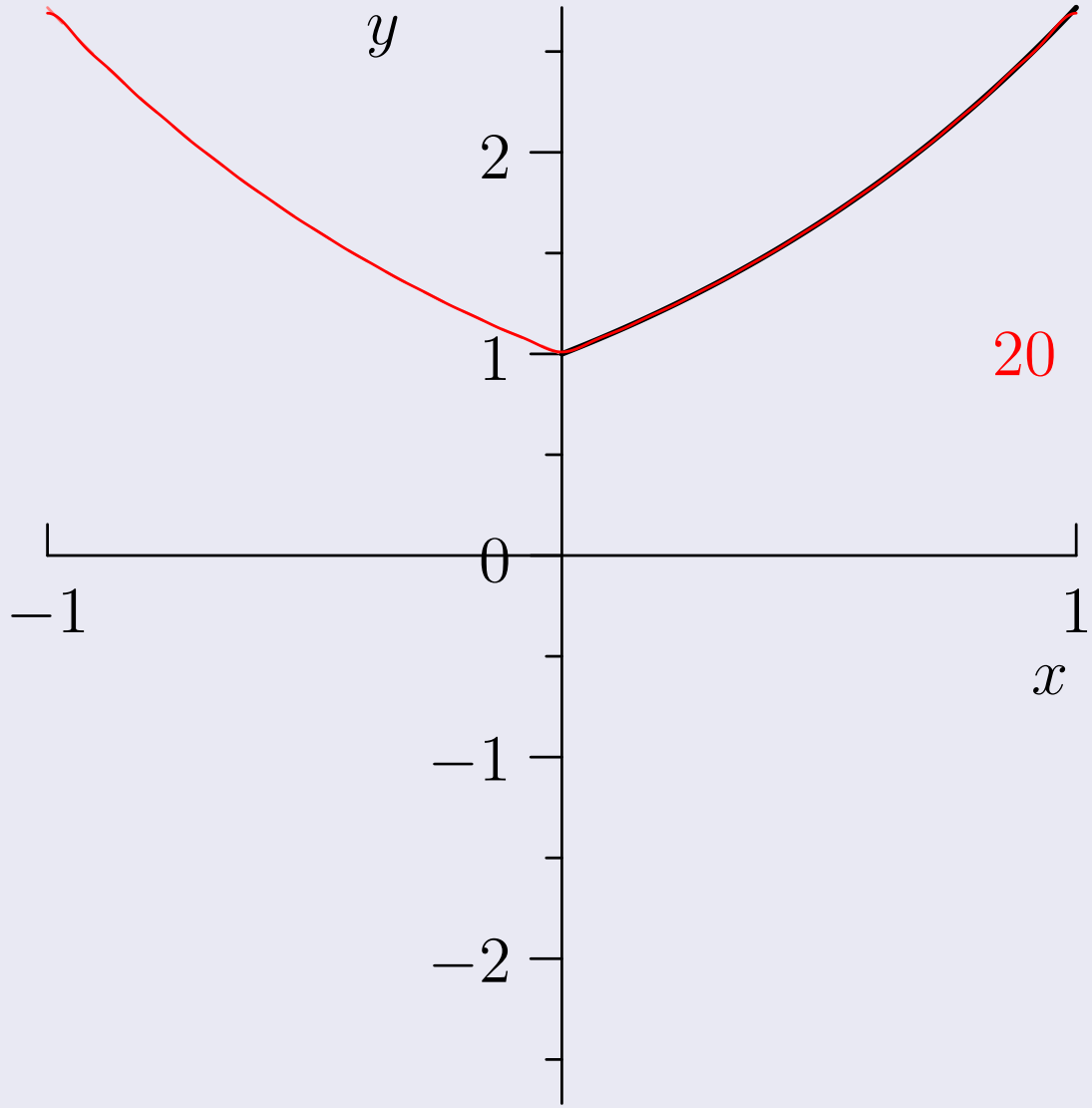

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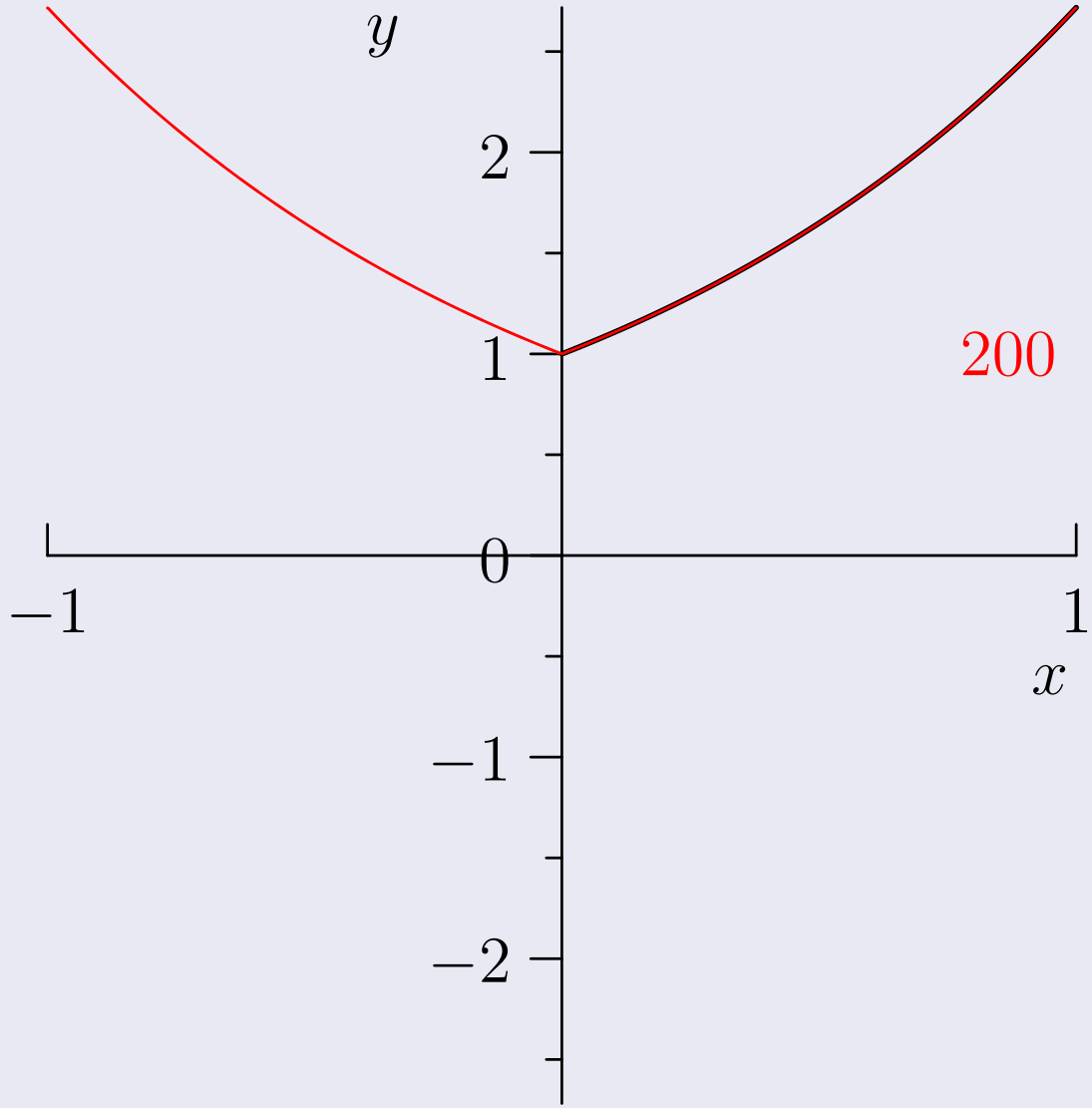

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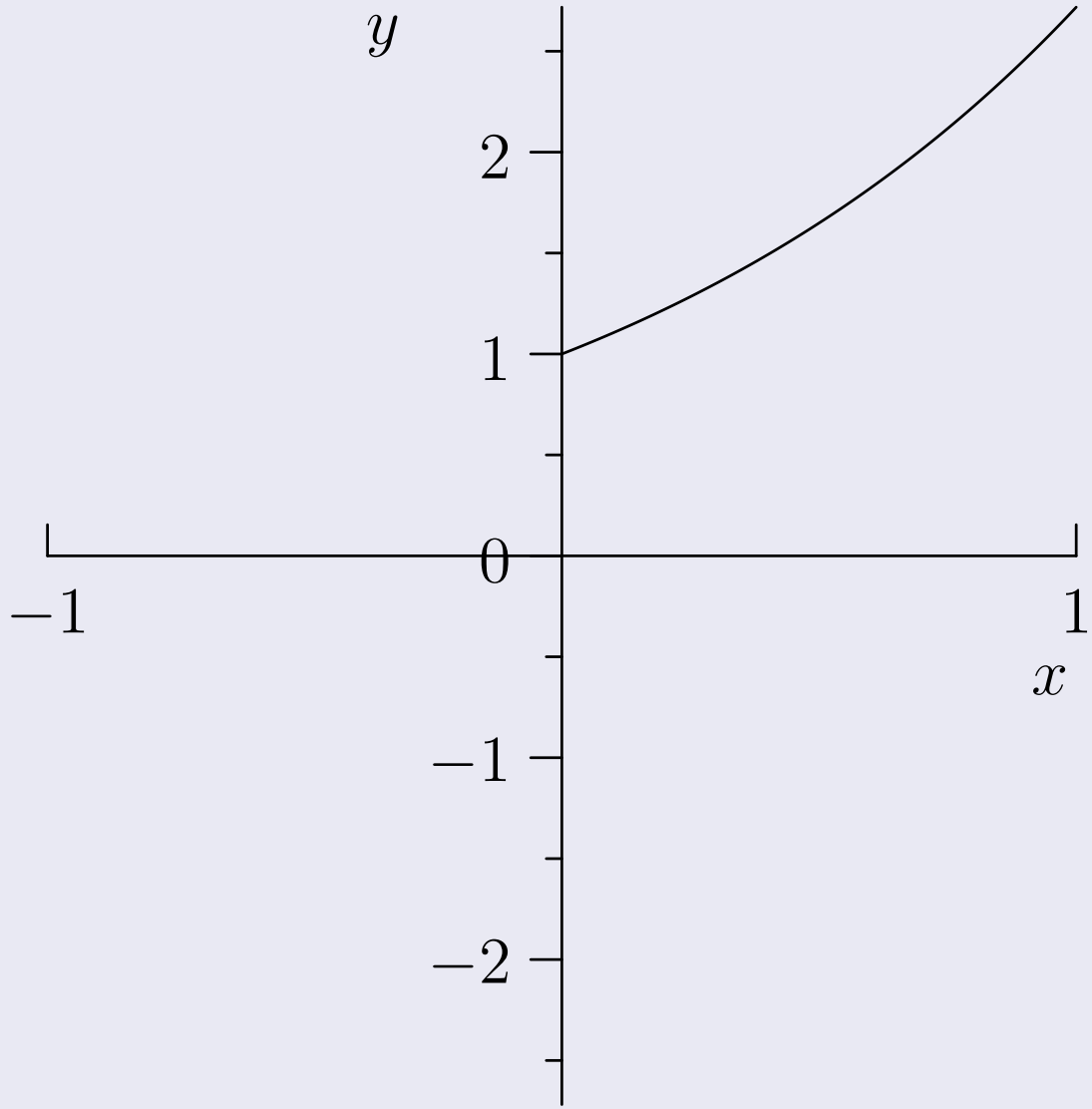

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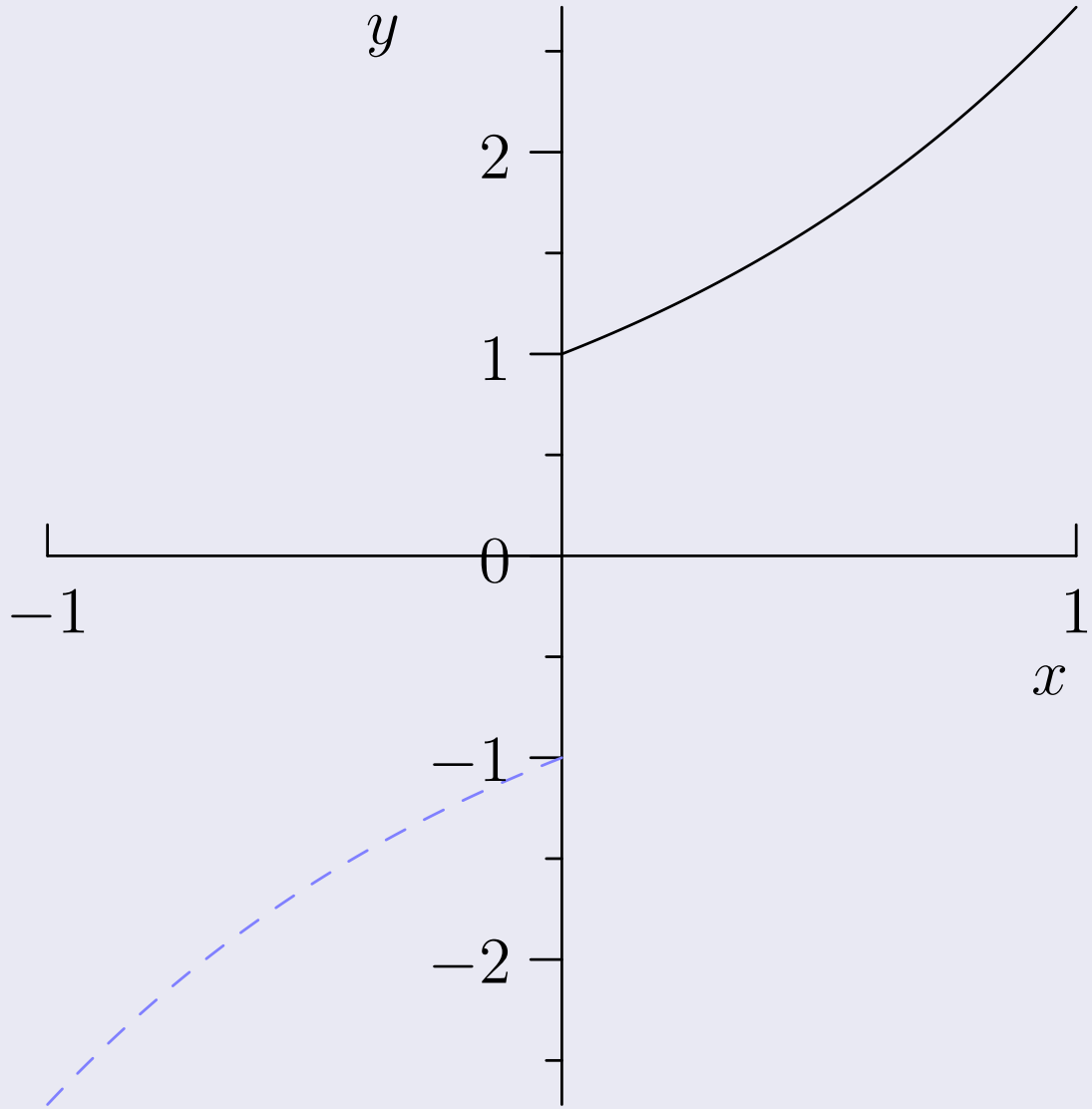

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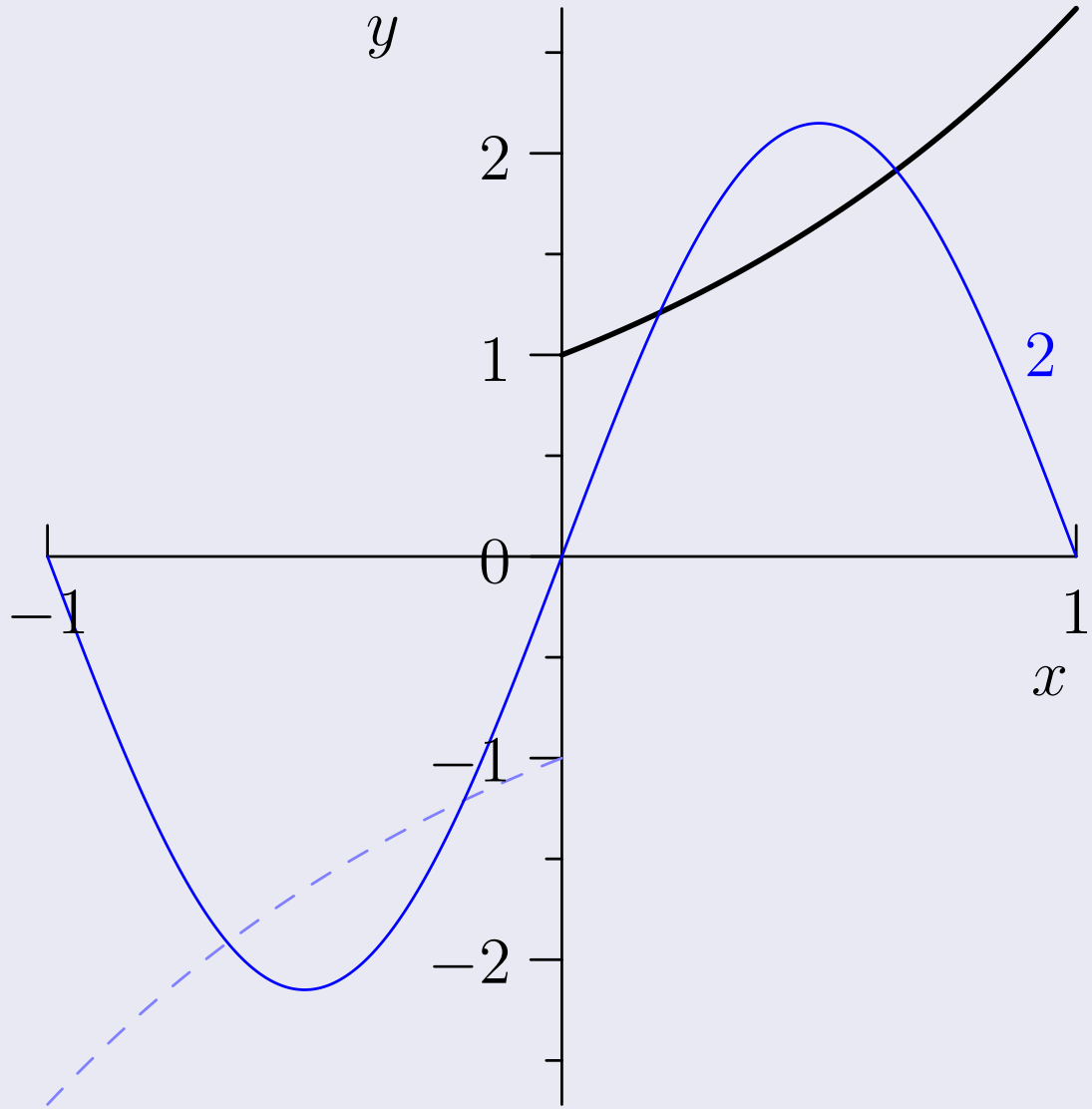

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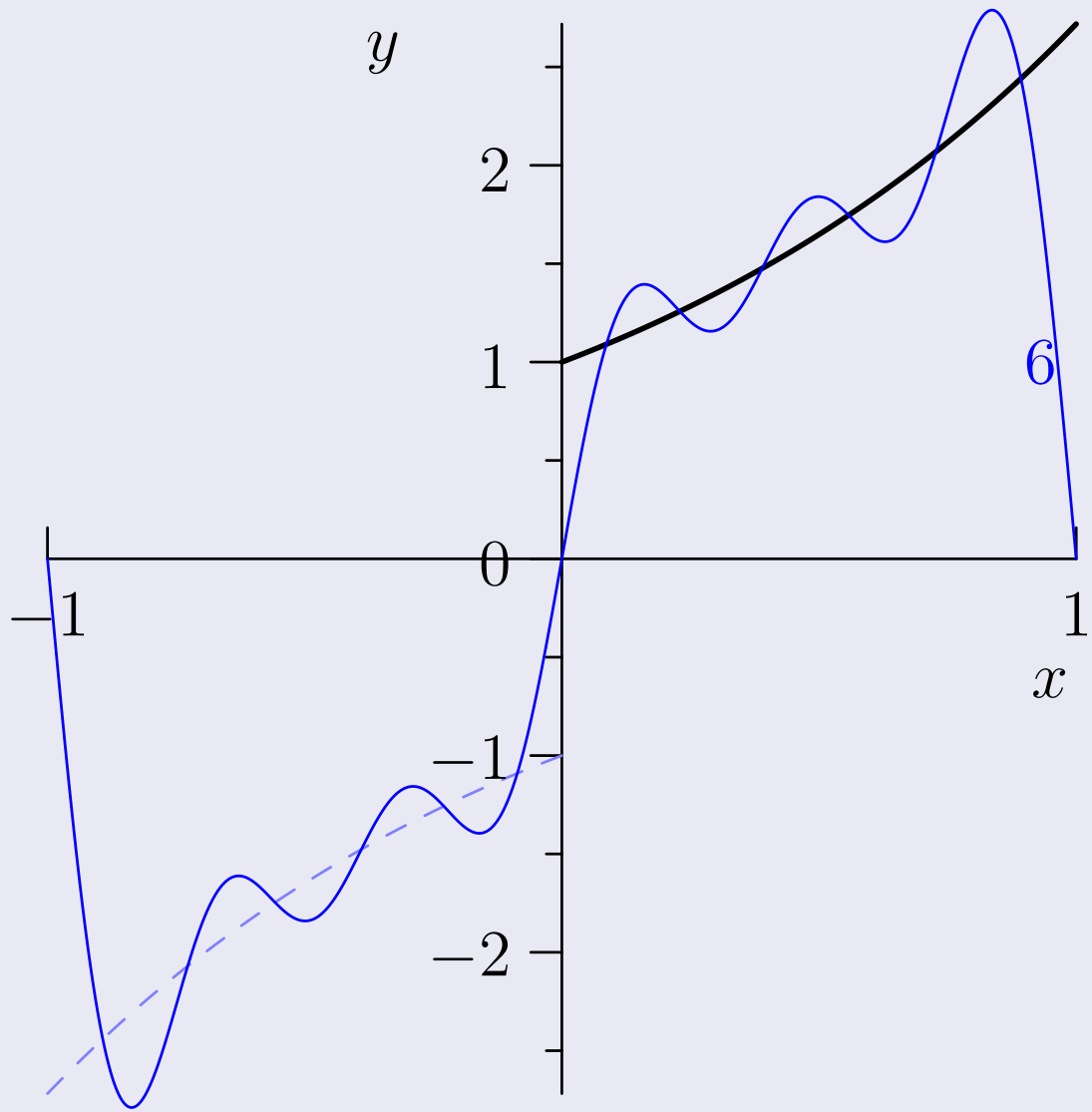

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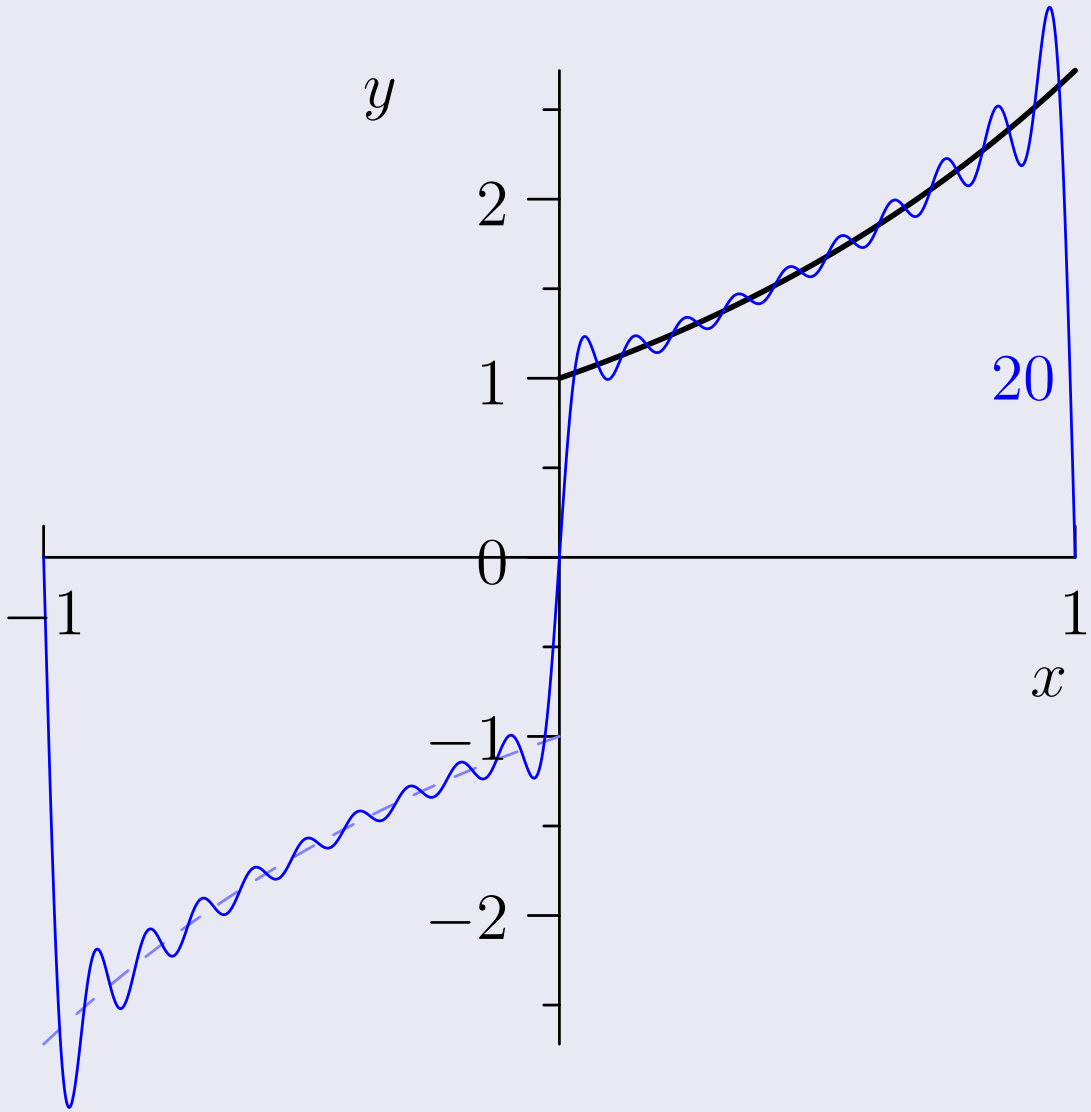

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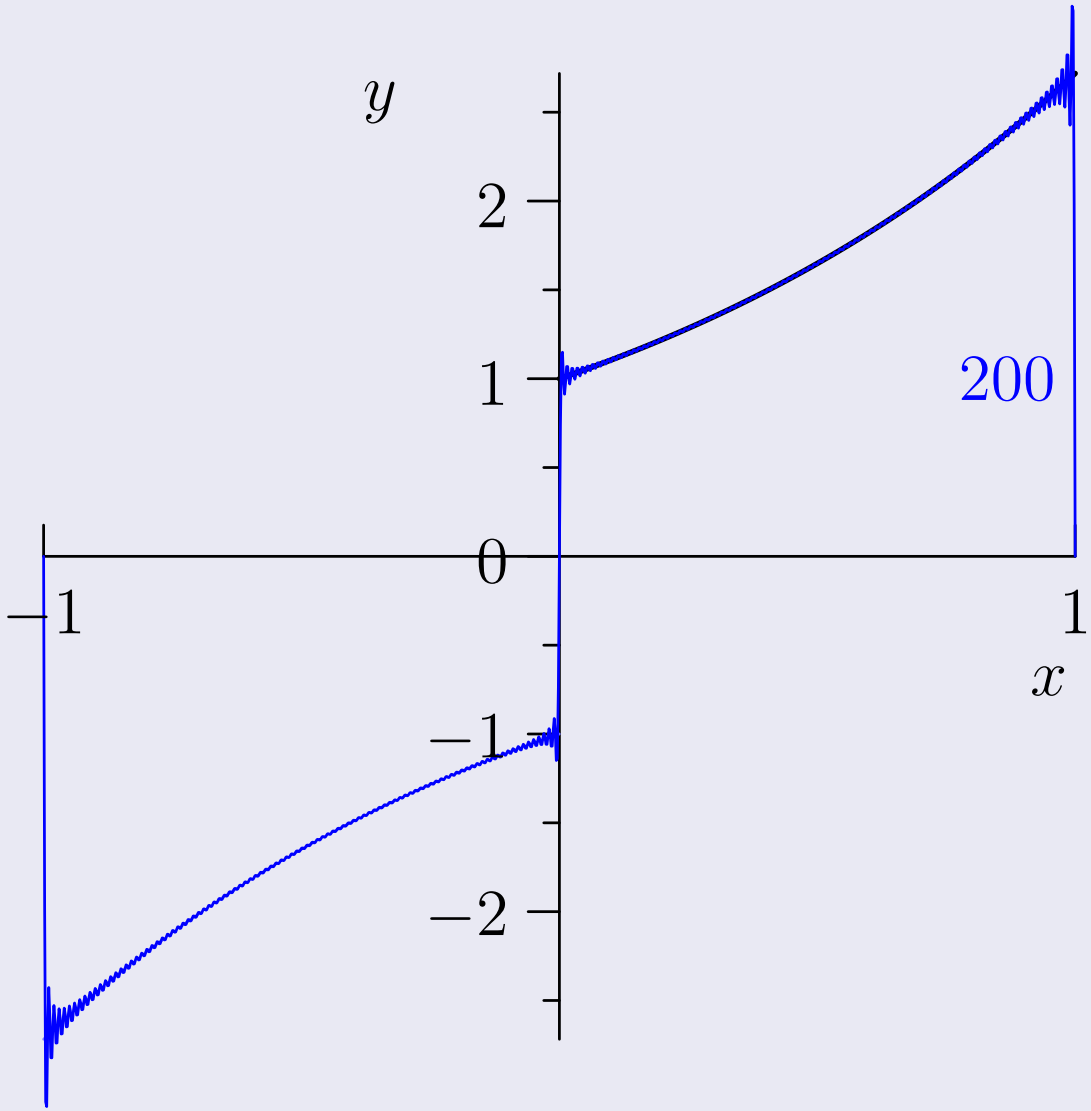

Translation Fourier ("Half-Range Fourier") Series

For f defined on $[0, L]$, the series

$$\frac{a_0}{2} + \sum_{m=1}^{\infty} \left(a_m \cos\left(\frac{2m\pi}{L}x\right) + b_m \sin\left(\frac{2m\pi}{L}x\right) \right)$$

converges* to $f(x)$, where

$$a_m = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2m\pi}{L}x\right)dx$$

$$b_m = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2m\pi}{L}x\right)dx$$