MATH 241 Practice Exam 3

This exam consists of 6 multiple-choice questions, 2 true-false questions, and one open-ended question. Show *all* your work. You will receive credit for a correct answer only if *your work is shown* and *your work supports your answer*. No credit will be given for correct answers which are not supported by work.

You will have 80 minutes to complete this exam.

The following table is for grading only. Do not write in it!

Problem	1	2	3	4	5	6	7.i	ii	8	TOTAL
Points										

1. Which of the following complex numbers has $\sin(z)$ in the first quadrant (excluding the axes)?

A. $z = i \ln 2$ B. $z = \pi + \left(\ln \sqrt{2}\right) i$ C. $z = 2\pi + \ln 2$

D. $z = \frac{\pi}{2} + \left(\ln \sqrt{2}\right) i$ E. $z = \frac{\pi}{4} + \frac{\pi}{2} i$ F. $\ln 2$

2. Let C be the ellipse $\frac{(x+1)^2}{9} + y^2 = 1$. What is

$$\oint_C \frac{1}{z(z-1)} dz$$

A. 0 B. πi C. $2\pi i$

D. $4\pi i$ E. $-4\pi i$ F. $\frac{\pi}{2}$

3. Let C be the polygonal curve with vertices (in order) $1-i$, $1+i$, $\frac{1}{2}$, $-1-i$. What is $\int_C \frac{1}{z} dz$?

A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$ C. $-\frac{\pi}{2}$

D. $\frac{1}{-i+1}$ E. $-\frac{\pi}{2}i$ F. $\frac{\pi}{4}i$

4. What is $\oint_C \frac{\exp(z)}{iz^4} dz$, where C is the square with vertices $1, i, -i, -1$?

A. 0 B. πi C. -2π D. $2\pi i$ E. $-\frac{\pi}{3}i$ F. $\frac{\pi}{3}$

5. Which of the following describes all complex numbers z with $|\exp(z)| = 1$?

A. $\{z||z| = 1\}$ B. $\{z|\cos^2 z + \sin^2 z = 1\}$ C. $\{z|\mathfrak{Im}(z) = 0\}$

D. $\{z|\mathfrak{Re}(z) = 0\}$ E. $\{z|\mathrm{Arg}(z) = \pi\}$ F. $\{z|\sin z \text{ is real}\}$

6. What is $\int_C \overline{z+1} dz$, where C is the parabola $y = x^2$, traversed from $x = 0$ to $x = 1$?

A. 0 B. $2 + \frac{4}{3}i$ C. $\frac{5}{6} + i$

D. $2\pi i$ E. $1 + 2i$ F. $1 - 2i$

7. TRUE or FALSE. For each of the following statements, indicate whether it is true (T) or false (F). Support your answers.

   i Let $u(x, y)$ and $v(x, y)$ be twice-differentiable real functions. If $u + iv$ is analytic, then so is $v + iu$.

   ii If $\int_C f(z)dz = 0$ for every simple closed contour in the domain D, then f is analytic.

8. Let $v(x, y) = \cos(x^2 - y^2)\sinh(2xy)$. Find a function $u(x, y)$ so that $u(x, y) + iv(x, y)$ is analytic.