Math 114 HW 5

due Thursday, 6/9

1. (Fall 2009) Suppose $z = f(x, y)$, where $x = g(s, t)$, $y = h(s, t)$. Suppose we know that
   $g(1,2) = 3, \quad g_s(1,2) = -1, \quad g_t(1,2) = 4,$
   $h(1,2) = 6, \quad h_s(1,2) = -5, \quad h_t(1,2) = 10,$
   $f_x(3,6) = 7, \quad f_y(3,6) = 8,$
   Find $\dfrac{\partial z}{\partial s} + \dfrac{\partial z}{\partial t}$ when $s = 1$ and $t = 2$.

   Hint : Use the chain rule.

   (a) 60 (b) -60 (c) 61 (d) -61 (e) 62 (f) -62

2. Let $T(x, y) = x^2 + y^2 - x - y$ be the temperature at the point (x, y) in the plane. A lizard sitting at the point $(1, 3)$ wants to increase his surrounding temperature as quickly as possible. In which direction should he move?

   (a) $\langle 1, 1\rangle$ (b) $\langle 1, 3\rangle$ (c) $\langle 1, 5\rangle$ (d) $\langle 1, 7\rangle$ (e) He should stay still. (f) none of the above

3. (Fall 2010) Let $f(x, y, z) = zx - xy^2$. At the point $(1, 1, 1)$, find the angle between the vector pointing in the direction of fastest increase of $f(x, y, z)$ and the x-axis.

   (a) -1 (b) $\frac{-1}{2}$ (c) 0 (d) $\frac{\pi}{6}$ (e) $\frac{\pi}{4}$ (f) $\frac{\pi}{3}$ (g) $\frac{\pi}{2}$

4. (Spring 2008) Let f be the function $f(x, y) = \ln(x + y)$ for $x + y > 0$. What is the maximum value of the directional derivative $D_u(f)$ of f at the point $(x, y) = (2, -1)$ as u ranges over all unit vectors in the x-y plane?

   (a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$ (e) $\ln(2)$ (f) 0 (g) none of the above

5. (Spring 2008) Find the equation of the tangent plane to the surface
   $$4x^4 + 2y^4 + z^4 = 22$$
   at the point $(1, 1, 2)$.

6. (Fall 2010) Consider the surface $z = x^2 + x + 2y^2$. At what point (x_0, y_0, z_0) is the tangent plane parallel to the plane $x + 4y + z = 0$?

7. (Fall 2011) Find the equation of the plane that is tangent to the surface
   $$\cos(y + x) - \sin(y + z) = \sin(z) - \cos(z)$$
   at the point $(\pi, \pi, 0)$. What is the y-coordinate of the point where this tangent plane intersects the y-axis?

8. (Spring 2009) The function $f(x, y) = x^4 + y^4 - 4xy + 1$ has how many local minima?
9. (Spring 2010) Consider the function $f(x, y) = -2x^3 + 3x^2 + 2y^2 - 4y$. Find the two critical points and determine their type (maxima/ minima/ saddle point).
10. (Spring 2013) Let $f(x, y) = x^3 - 3xy + y^2$. Find the local minimum of f.